

1. (20pts) A 10-member board holds an election for a chairperson. The candidates are Demoya, Ferguson, MacArthur and Hearn. Their preference rankings and approval votes are as follows:

Number of voters:	1	3	1	2	1	2
Demoya	1√	1√	2	3	2√	3√
Ferguson	2√	3	1√	4	3√	4
MacArthur	3√	2√	4	1√	4√	2√
Hearn	4	4	3	2√	1√	1√

- Who wins the vote in a plurality election?
- Who wins the vote in a plurality election, followed by a runoff of the first two finishers?
- Who is the Condorcet winner, if any?
- Who is the winner using Borda's method? Perform the check on the sum of Borda points.
- Who is the winner using the approval method?
- In the Borda election, could the two voters who ranked MacArthur first achieve a preferable outcome by voting strategically, assuming all the other members voted as shown?

a)

	no of votes
D.	4 ← Demoya wins plurality
F.	1
M.	2
H.	3

d) Borda counts

$$\begin{aligned}
 D & 4(1+3) + 3(1+1) + 2(2+2) = 16 + 6 + 8 = 30 \\
 F & 4 \cdot 1 + 3 \cdot 1 + 2 \cdot 4 + 1 \cdot 4 = 4 + 3 + 8 + 4 = 19 \\
 M & 4 \cdot 2 + 3 \cdot (2+2) + 2 \cdot 1 + 1 \cdot (1+3) = 8 + 12 + 2 + 4 = 27 \\
 H & 4 \cdot 3 + 3 \cdot 2 + 2 \cdot 1 + 1 \cdot (1+3) = 12 + 6 + 2 + 4 = 24
 \end{aligned}$$

100

b) Runoff between Demoya & Hearn:

	votes
D	5
H	5

> They tie

Demoya wins

$$(H+3+4) \cdot 10 = 100$$

votes

c) Head-to-head comparisons

D 9	F 3	Demoya is the weak
F 1	M 7	
D 6	F 5	Condorcet winner
M 4	H 5	
D 5	M 6	
H 5	H 4	

e) Approval: D 7
F 3
M 9
H 5

MacArthur wins

If the two voters change their vote, D will have at least 28.

	two voters contribute	w/o the voters
f) D	30	-4 = 26
F	19	-2 = 17
M	27	-8 = 19
H	24	-6 = 18

If they rank M first, M has 27, if they rank H first, H has 26. In either case, not enough to defeat D.