Fall '04/MAT 117/Final Exam **Name:** *Show all your work.*

$F = P(1+rt)$ $F = P\left(1+\frac{r}{n}\right)^{nt}$ $F = D\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = R\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $APY = \left(1+\frac{r}{n}\right)^{n}-1$

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $P(E) = \frac{b}{a+b}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

angle = (relative frequency) $\cdot$ 360° $Z = \frac{X-\mu}{\sigma}$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad \sigma = \sqrt{\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \qquad \sigma = \sqrt{\frac{f_1(x_1-\mu)^2 + f_2(x_2-\mu)^2 + \cdots + f_n(x_n-\mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

1. (11pts) The ages of all Boy Scouts in a small town are summarized in the table below.
a) Find the median age.
b) Find the mean age.
c) Compute the relative frequencies for each age.
What should the relative frequencies add up to?
d) Draw a histogram showing relative frequencies.

Age	Frequency	Relative Freq.
12	21	0.28
13	24	0.32
14	13	0.17
15	8	0.11
16	5	0.07
17	4	0.05
	75	should add up to 1

a) $\underbrace{12,\dots,12}_{21}, \underbrace{13,\dots,13}_{24}, \underbrace{14,\dots,14}_{13}, \underbrace{15,\dots,15}_{8}, \underbrace{16,\dots,16}_{5}, \underbrace{17,\dots,17}_{4}$

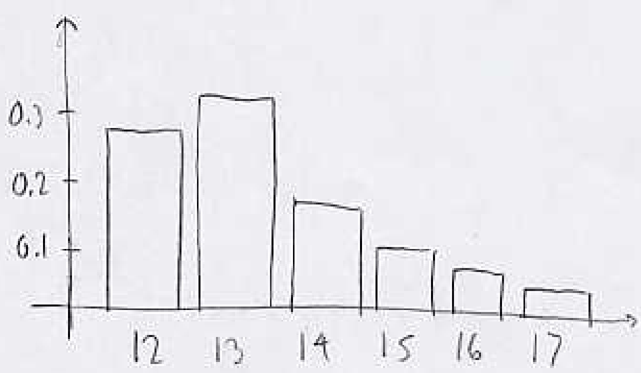
here

Need 38th number

median = 13

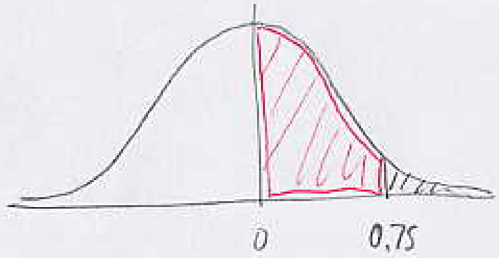
b) $\mu = \frac{21 \cdot 12 + 24 \cdot 13 + 13 \cdot 14 + 8 \cdot 15 + 5 \cdot 16 + 4 \cdot 17}{75}$

$= 13.52$



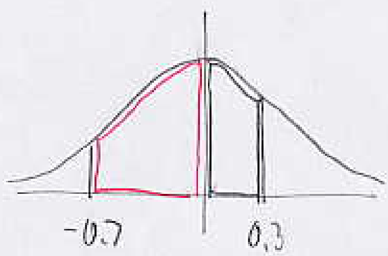
2. (4pts) If Z is a random variable for a standard normal distribution, compute the probability below. Draw a picture showing which area you are computing.

$P(0.75 \leq Z) =$ $0.5 - \text{red area} = 0.5 - 0.2734 = 0.2266$



3. (6pts) Suppose the scores on a test are normally distributed with mean 72 and standard deviation 10. Find the probability that a random student scored between 65 and 75.

$$P(65 \leq X \leq 75) = P\left(\frac{65-72}{10} \leq Z \leq \frac{75-72}{10}\right) = P(-0.7 \leq Z \leq 0.3)$$



$= \text{red area} + \text{black area} = 0.2580 + 0.1179$

$= 0.3759$

4. (5pts) Suppose three candidates — Godfrey, Smith and Mawson — are running in an election that is to be decided by plurality followed by a runoff of the two top finishers. The results of a the plurality election are: Godfrey 465, Smith 435 and Mawson 100. What is the smallest number of Mawson supporters that need to vote for Godfrey in order for Godfrey to win the election?

Total voters $= 465 + 435 + 100 = 1000$

To win runoff, Godfrey needs 501 votes, thus,

an additional $501 - 465 = 36$ votes.

5. (13pts) A pool of critics ranked three modern-day movie directors. Their rankings are shown in the table.

Percent of votes:	13	21	17	12	24	13
Ridley Scott *(Gladiator, Blade Runner)*	1	1	2	3	2	3
Quentin Tarantino *(Pulp Fiction, Kill Bill)*	2	3	1	1	3	2
Steven Soderbergh *(Traffic, Ocean's Eleven)*	3	2	3	2	1	1

a) Which director wins in a plurality election?
b) Which director wins in a plurality election, followed by a runoff of the first two finishers?
c) Which director wins using the Borda method?
d) Perform the check on the sum of Borda points.
e) Can the critics who ranked Tarantino first and Soderbergh second obtain a preferable outcome if they voted strategically, assuming all the other critics voted as shown in the table?

a) Scott $21+13=34$

Tara. $17+12=29$

Soder. $24+13=37$ ← wins.

b) Scott. $34+17=51$ ← wins

Soder. $37+12=49$

c) Scott: $3\cdot 34+2\cdot 41+1\cdot 25=209$

Tara: $3\cdot 29+2\cdot 26+1\cdot 45=184$

Soder: $3\cdot 37+2\cdot 33+1\cdot 30=207$

total 600

d) Sum of Borda pts should be $100\cdot 6$.

e)

	without the 12% vote	pts. added if the 12% vote 3 2 1	Total
Scott	197	12	209
Tara.	148	24	172
Soder.	183	36	219 ← wins

If those voters rank Soder. 1st, Tarantino 2nd, Soder. wins, which is a preferable outcome for them.

6. (4pts) How long does it take for \$1000 to grow to \$1500 in a simple interest account yielding an annual interest rate of 5%?

$$F = P(1+rt)$$

$$1500 = 1000(1+0.05t) \quad | \div 1000$$

$$1.5 = 1 + 0.05t \quad | -1$$

$$0.5 = 0.05t \quad | \div 0.05$$

$$\frac{0.5}{0.05} = t$$

$$t = 10 \text{ yrs}$$

7. (4pts) What is the future value, after 3 years, of a one-time deposit of \$2400 into an account bearing 4% interest compounded weekly?

$$F = P\left(1+\frac{r}{n}\right)^{nt}$$

$$F = 2400\left(1+\frac{0.04}{52}\right)^{52\cdot 3}$$

$$= 2705.87$$

8. (7pts) Count Dracula wishes to build a new tomb for \$55,000. Suppose he can get a 15-year loan with interest rate 2%, compounded monthly. (Vampires can get low interest rates because banks know they'll be dead for a while — check with a loan officer :)

a) What is his monthly payment?

b) What is the balance on the loan after 8 years?

a) $$P = R\,\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$$

$$55000 = R \cdot \frac{1-\left(1+\frac{0.02}{12}\right)^{-180}}{\frac{0.02}{12}}$$

$$\frac{55000}{155.398} = R$$

$$R = 353.93$$

b) Balance on loan = present value of remaining 7 yers of payments.

$$P = 353.93\,\frac{1-\left(1+\frac{0.02}{12}\right)^{-12\cdot 7}}{\frac{0.02}{12}}$$

$$P = 27{,}721.30$$

9. (7pts) A coin is tossed 3 times.
a) How many outcomes does this experiment have?
b) What is the probability of getting exactly one head?
c) What is the probability of getting at most two tails?

a) HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

8 outcomes

b) $P(\text{exactly one head}) = \frac{3}{8}$

c) $P(\text{at most two tails}) = 1 - P(\text{not (at most two tails)})$

$= 1 - P(\text{three tails})$

$= 1 - \frac{1}{8} = \frac{7}{8}$

10. (5pts) In a class of 35 students, 20 use calculators, 18 use computers, and 8 use both calculators and computers What is the probability that a randomly chosen student
a) uses a calculator or a computer?
b) uses neither a calculator or a computer?

a) $P(\text{calc or comp}) = P(\text{calc}) + P(\text{comp}) - P(\text{calc. and comp.})$

$= \frac{20}{35} + \frac{18}{35} - \frac{8}{35} = \frac{30}{35} = \frac{6}{7}$

b) $P(\text{neither calc nor comp.}) = P(\text{not(calc. or comp.)})$

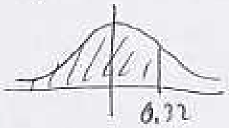
$= 1 - \frac{6}{7} = \frac{1}{7}$

11. (4pts) A road has two traffic lights that operate independently of each other. The first traffic light is green 43% of the time, the second is green 75% of the time. If a driver's route takes her through both traffic lights, what is the probability that she has to stop at at least one of the traffic lights?

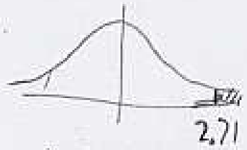
$$P(\text{red at at least one}) = 1 - P(\text{green on both})$$
$$= 1 - P(\text{green on first}) \cdot P(\text{green on second})$$
$$= 1 - 0.43 \cdot 0.75$$
$$= 0.6775$$

Bonus. (7pts) A survey has found that weights of a large population of employed men are normally distributed with mean 192lbs and standard deviation 25. The same survey found that their salaries were normally distributed with mean \$41,000 and standard deviation \$7,000. Assuming that weight and salaries are independent of each other, what is the probability that a randomly chosen man has weight less than 200lbs or a salary greater than \$60,000? (Hint: use your knowledge of probability AND statistics here.)

$$P(\text{weighs} \leq 200 \text{ OR has salary} \geq 60K)$$
$$= P(\text{weighs} \leq 200) + P(\text{salary} \geq 60k) - P(\text{weighs} \leq 200 \text{ AND salary} \geq 60k)$$
$$= \quad " \qquad " \quad - P(\text{weighs} \leq 200)\, P(\text{salary} \geq 60k)$$
$$= P(X_1 \leq 200) + P(X_2 \geq 60{,}000) - P(X_1 \leq 200)\, P(X_2 \geq 60{,}000)$$
$$= P\left(Z_1 \leq \frac{200-192}{25}\right) + P\left(Z_2 \geq \frac{60{,}000-41{,}000}{7{,}000}\right) + P(\_)\cdot P(\_)$$
$$= P(Z_1 \leq 0.32) + P(Z_2 \geq 2.71) + P(Z_1 \leq 0.32)\cdot P(Z_2 \geq 2.71)$$



$0.5 + 0.1255$



$0.5 - 0.4966$

$$= 0.6255 + 0.0034 - 0.6255 \cdot 0.0034$$
$$= 0.6268$$