Fall '04/MAT 117/Exam 3 **Name:** Solution *Show all your work.*

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $\quad P(E) = \frac{b}{a+b}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ $\quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

1. (2pts) Suppose the town Murkville had 186 cloudy days last year. Based on this information, what is the empirical probability that it will be cloudy in Murkville on a random day?

$$P(\text{cloudy}) = \frac{186}{365} = 0.51$$

2. (2pts) If 37.6% of burglaries are committed by someone who knows the victim, what is the probability that a random victim was burglarized by a stranger?

$$P(\text{burglary by stranger}) = 1 - P(\text{burglary by acquaintance})$$
$$= 1 - 0.376 = 0.624$$

3. (6pts) Two dice are rolled.

a) How many outcomes does this experiment have?

b) List the outcomes for which the sum on the dice is 8.

c) What is the probability of getting a sum of 8 on one roll of two dice?

a) $36 = 6 \cdot 6$

b) $\left.\begin{matrix} 2,6 \\ 3,5 \\ 4,4 \\ 5,3 \\ 6,2 \end{matrix}\right\}$ 5 outcomes

c) $P(\text{sum is } 8) = \frac{5}{36} = 0.139$

4. (3pts) If a random card is drawn from a deck, the odds against this card being a spade are 3 to 1.

spade : 13

not spade : 39

$\frac{39}{13} = \frac{3}{1}$

5. (3pts) If the odds against winning a \$5 prize in a lottery game are 13 to 2, what is the probability of winning \$5 in the game?

$$P(E) = \frac{2}{13+2} = \frac{2}{15} = 0.133$$

6. (4pts) Suppose the house odds on the horse Seven Legs are 3 to 7. If you think the horse's chances of winning are 70%, is this a fair bet? Hint: compute true odds against winning.

$$\text{True odds} = \frac{1-0.7}{0.7} = \frac{0.3}{0.7} = \frac{3}{7}$$

Since true odds = house odds, it's a fair bet.

7. (6pts) In a neighborhood of 68 homes, 51 of the homes have fences, 29 have sprinkling systems and 20 have both fences and sprinkling systems. If a home is randomly selected, what is the probability that

a) it has a fence or a sprinkling system?

b) it has neither a fence nor a sprinkling system?

a) $P(\text{fence or sprinkler}) = P(\text{fence}) + P(\text{sprinkler}) - P(\text{fence and sprinkler})$

$$= \frac{51}{68} + \frac{29}{68} - \frac{20}{68} = \frac{51+29-20}{68} = \frac{60}{68} = \frac{15}{17} = 0.882$$

b) $P(\text{neither fence nor sprinkler}) = P(\text{not (fence or sprinkler)})$

$$= 1 - \frac{15}{17} = \frac{2}{17} = 0.118$$

8. (7pts) Suppose a manufacturing process requires two steps. The first step is successful 97% of the time, the second is successful 95% of the time. Assume the success of the two steps is independent.
a) What is the probability that both steps are completed successfully?
b) What is the probability that neither step was performed successfully?

a) $P(\text{1st succ. and 2nd succ.}) = P(\text{1st succ.}) \cdot P(\text{2nd succ.})$
$= 0.97 \cdot 0.95 = 0.9215$

b) $P(\text{1st fail and 2nd fail}) = P(\text{1st fail}) \cdot P(\text{2nd fail})$
$= 0.03 \cdot 0.05$
$= 0.0015$

9. (9pts) A class has 8 boys and 10 girls. If two children are selected at random, what is the probability
a) that they are both girls?
b) that the second child is a boy, given that the first is a girl?
c) that the second child is a boy?

a) $P(\text{1st girl and 2nd girl})$
$= P(\text{1st girl}) \cdot P(\text{2nd girl} \mid \text{1st girl}) = \frac{10}{18} \cdot \frac{9}{17} = \frac{5}{9} \cdot \frac{9}{17} = \frac{5}{17} \approx 0.294$

b) $P(\text{2nd boy} \mid \text{1st girl}) = \frac{8}{17} \approx 0.471$

c) $P(\text{2nd boy}) = \frac{8}{18} = \frac{4}{9} \approx 0.444$

10. (8pts) Suppose a multiple-choice exam has four possible answers for each question, only one of them correct. You get 5 points for each correct answer, lose 2 points for each incorrect answer and nothing if you leave the question unanswered.
a) What is the expected point value of a random guess on this exam?
b) What is the expected point value if you can eliminate one of the answers as incorrect and choose a random answer from the remaining three?
c) Assuming you can always eliminate one answer and choose a random answer from the remaining ones, how many points would you expect to get on a test with 50 questions?

a) exp. value = P(correct) $\cdot 5$ + P(incorrect) $\cdot (-2)$

$$= \frac{1}{4}\cdot 5 + \frac{3}{4}\cdot(-2) = \frac{5}{4} - \frac{6}{4} = -\frac{1}{4}$$

b) exp value $= \frac{1}{3}\cdot 5 + \frac{2}{3}\cdot(-2) = \frac{5}{3} - \frac{4}{3} = \frac{1}{3}$

c) You'd expect to have $50\cdot\frac{1}{3} = \frac{50}{3} = 16.667$ pts.

Bonus. (5pts) Woong and Arim are separately asked to choose between an apple and an orange. They will get the snack only if they both name the same choice. If Woong names apple 40% of the time and orange 60% of the time, and Arim names apple 55% of the time and orange 45% of the time, what is the probability that they get a snack?

P(get snack) = P((W. apple and A. apple) or (W. orange and A. orange))

mutually exclusive

= P(W. apple and A. apple) + P(W. orange and A. orange)

independent independent

= P(W. apple) · P(A. apple) + P(W. orange) · P(A. orange)

$= 0.4 \cdot 0.55 + 0.6 \cdot 0.45 = 0.49$