$$F = P(1+rt) \quad F = P\left(1+\tfrac{r}{n}\right)^{nt} \quad F = D\frac{\left(1+\tfrac{r}{n}\right)^{nt}-1}{\tfrac{r}{n}} \quad P = R\frac{1-\left(1+\tfrac{r}{n}\right)^{-nt}}{\tfrac{r}{n}} \quad APY = \left(1+\tfrac{r}{n}\right)^{n}-1$$

1. (8pts) Solve the following equations. Round the answer to 8 decimal places.

$3^x = 45$ $| \log$

$\log 3^x = \log 45$

$x \log 3 = \log 45$

$x = \dfrac{\log 45}{\log 3} = 3.46497352$

$(1+x)^5 = 2.4$ $|^{\frac{1}{5}}$

$\left((1+x)^5\right)^{\frac{1}{5}} = 2.4^{\frac{1}{5}}$

$1+x = 2.4^{\frac{1}{5}}$

$x = 2.4^{\frac{1}{5}} - 1$

$= 0.19135790$

2. (4pts) What is the future value of $1000 deposited for 18 months in an account bearing simple interest of 13%?

$F = 1000\left(1 + 0.13 \cdot 1.5\right) = 1,195.00$

$18 \text{ months} = 1.5 \text{ years}$

3. (6pts) A man borrows $200 from a pawn shop that he repays with $260 after 3 months. What simple annual interest rate has he been charged?

$r = ? \quad , \quad t = 3 \text{ months} = \tfrac{1}{4} \text{ year}$

$260 = 200\left(1 + r \cdot \tfrac{1}{4}\right)$

$\dfrac{260}{200} = 1 + \tfrac{r}{4}$

$1.3 - 1 = \tfrac{r}{4}$

$0.3 = \tfrac{r}{4} \quad | \cdot 4$

$1.2 = r$

Rate is 120% per year.

4. (6pts) What is the future value, after 5 years, of $2000 deposited into an account bearing 7% interest compounded daily?

$$F = 2000 \left(1 + \frac{0.07}{365}\right)^{365 \cdot 5} = 2838.04$$

5. (8pts) Jennifer wishes to save $20,000 for a down payment on a house. She can get a savings account bearing 4% compounded quarterly.
a) How much should she deposit at the end of every quarter in order to have $20,000 after 4 years?
b) Using your answer from a) (instead of the formula), how much should she deposit quarterly under the same terms if she wishes to have $30,000 after 4 years?

a) $$20000 = D \frac{\left(1 + \frac{0.04}{4}\right)^{4 \cdot 4} - 1}{\frac{0.04}{4}}$$

$$20000 = D \cdot \frac{1.01^{16} - 1}{0.01}$$

$$20000 = D \cdot 17.25786449$$

$$\frac{20000}{17.25\ldots} = D$$

$$D = 1158.89$$

b) Since $30000 = 1.5 \cdot 20000$

the payment is

$$1.5 \cdot 1158.89 = 1738.34$$

6. (10pts) The Bullynator was whacked so hard in the last exam that he landed in this one. Having fallen less than safely, he also incurred a $25,000 emergency room bill, for which he got a 10-year loan from his bank at 6% interest, compounded monthly.

a) What is his monthly payment?

b) What is the balance on the loan after 4 years?

a)
$$25000 = R \cdot \frac{1 - \left(1 + \frac{0.06}{12}\right)^{-12 \cdot 10}}{\frac{0.06}{12}}$$

$$25000 = R \cdot \frac{1 - (1.005)^{-120}}{0.005}$$

$$25000 = R \cdot 90.07345333$$

$$\frac{25000}{90.073\ldots} = R$$

$$R = 277.55$$

b) Balance = present value of remaining 6 years of payments

$$P = 277.55 \frac{1 - (1.005)^{-12 \cdot 6}}{0.005}$$

$$P = 16747.23$$

7. (8pts) Suppose you can deposit \$100 every month into an account bearing 9% interest compounded monthly. How long will it take you to save \$5,000?

$$5000 = 100 \frac{\left(1 + \frac{0.09}{12}\right)^{12t} - 1}{\frac{0.09}{12}} \quad \big| \div 100$$

$$\frac{5000}{100} = \frac{(1.0075)^{12t} - 1}{0.0075} \quad \big| \cdot 0.0075$$

$$50 \cdot 0.0075 = 1.0075^{12t} - 1 \quad \big| + 1$$

$$1.375 = 1.0075^{12t} \quad \big| \log$$

$$\log 1.375 = \log 1.0075^{12t}$$

$$\log 1.375 = 12t \log 1.0075 \quad \big| \div 12 \log 1.0075$$

$$t = \frac{\log 1.375}{12 \log 1.0075}$$

$$= 3.55 \text{ years}$$

Bonus. (5pts) Bank of Shanghai is offering a savings account bearing 4.2% compounded monthly. Competing Bank of Beijing is offering a savings account bearing 4.1% compounded ~~daily.~~ hourly. What is the better deal? (Hint: consider a deposit for 1 year).

Compute value of \$ deposited with either bank.

Shanghai: $F = 1 \cdot \left(1 + \frac{0.042}{12}\right)^{12} = 1.042818007$ ← more, so a better deal.

Beijing: $F = 1 \cdot \left(1 + \frac{0.041}{365.24}\right)^{365.24} = 1.041852006$