

1. (11pts) A poll of members of a community asked people about their opinion of a new speed limit for a local highway. Their preference rankings broke down into the following percentages:

Percent of voters:	28	20	13	39
55 mph	1	2	3	3
60 mph	2	1	1	2
65 mph	3	3	2	1

- a) Which option wins the vote in a plurality election?  
 b) Which option wins in a plurality election, followed by a runoff of the first two finishers?  
 c) Is any option a Condorcet winner or a weak Condorcet winner?

a) 55 28%

60 33%

65 39%

65 wins plurality

b) Runoff between 60 and 65

60:  $33 + 28 = 61\%$

65:  $39 + 0 = 39\%$

c)  $\left\{ \begin{array}{l} 55: 28 = 28 \\ 60: 20 + 13 + 39 = 72 \text{ w} \end{array} \right.$

$\left\{ \begin{array}{l} 55: 28 + 20 = 48 \\ 65: 13 + 39 = 52 \text{ w} \end{array} \right.$

$\left\{ \begin{array}{l} 60: 28 + 20 + 13 = 61 \\ 65: 39 \end{array} \right.$

60 is the Condorcet winner

2. (5pts) If 75 votes are cast, what is the smallest number of votes a winning candidate can have in a four-candidate race that is decided by plurality? Justify your answer.

A candidate wins with the fewest votes if all votes are distributed almost evenly.

$$\frac{75}{4} = 18, \text{ remainder } 3.$$

A candidate with 19 votes will not win, since another one will have at least 19.

Thus, 20 is the fewest votes with which a candidate can win.

3. (13pts) A baseball team wants to select a nickname for the team. Their preference rankings are as follows:

	Number of votes:						(total 19)
	1	7	3	4	1	3	
Eagles	1	1	2	3	2	3	
Panthers	2	3	1	1	3	2	
Wasps	3	2	3	2	1	1	

- Which nickname wins using the Borda method?
- Perform the check on the sum of Borda points.
- Can the three players who ranked Wasps first and Panthers second obtain a preferable outcome if they voted strategically, assuming all the other players voted as shown in the table?

a)  $E \quad 3 \cdot (1+7) + 2 \cdot (3+1) + 1 \cdot (4+3) = 24 + 8 + 7 = 39 \leftarrow \text{Eagles wins}$

$P \quad 3(3+4) + 2 \cdot (1+3) + 1 \cdot (7+1) = 21 + 8 + 8 = 37$

$W \quad 3(1+3) + 2 \cdot (7+4) + 1 \cdot (1+3) = 12 + 22 + 4 = 38$

b) sum of pts = 114

Need to get  $19 \cdot 6 = 114$

$\uparrow$  voters      $\uparrow$  pts. per ballot

c) Preferable outcome is W or P.

	3 voters contribute	pts w/o the 3 voters	if the 3 voters rank	scores are
E 39	3	36	3	39
P 37	6	31	1	40 $\leftarrow$ P wins
W 38	9	29	2	35

It is not possible to achieve a W win, since E has at least 39, but by ranking P first, the voters can get a P win.

4. (11pts) All is not well in Toontown. The Bullynator showed up uninvited and is causing all sorts of trouble. The cartoon characters banded together and are deciding how to get rid of him. Their options are: disintegrate him with a ray gun; whack him so hard that he flies out of the movie frame; let Pepe Le Pew loose on him; and wrap him in a package and ship him to Cuba. The 'toons voted by approval:

		Number of votes:						
		11	7	8	10	4	6	2
	Ray gun		✓	✓			✓	✓
my vote	✓ Whack	✓	✓		✓	✓	✓	✓
	✓ Pepe	✓		✓		✓	✓	✓
	Cuba		✓		✓	✓	✓	✓

- Which method wins an approval vote?
- Which voters have no influence on the outcome?
- Assuming the 4 'toons who approved of Whack, Pepe and Cuba prefer Pepe to Cuba to Whack, could they alter their approval vote in order to get a preferable outcome? Assume the other votes stay the same.
- For a fun poll, mark your approval vote on the left of the table.

$$\begin{aligned} a) \quad R & 7+8+5+2 = 23 \\ W & 11+7+10+4+2 = 34 \\ P & 11+8+4+6+2 = 31 \\ C & 7+10+4+6+2 = 29 \end{aligned}$$

b) The voters who approve of all candidates do not influence the election.

c) If P or C are preferred to W, by not approving W, the 4 voters can lower W's count to 30, so P would win. C can't win by a change of vote of these voters since W will have at least 30 and C has at most 22

5. (10pts) Suppose 105 votes are cast in a plurality election between candidates Diaz, Fenton and Hung. After the first 70 votes are counted, the tally is: Diaz 19, Fenton 24, Hung 27. Answer and justify:

- a) What is the minimal number of remaining votes Hung needs to be assured of a win?  
 b) What is the minimal number of votes Diaz needs to be assured of a win?

35 votes remain.

a) In the worst-case scenario

Fenton catches up with 3 votes

Of the remaining 32 votes,

Hung needs 17 to be assured

of a win. So,  $\textcircled{17}$ .

b) Diaz first needs 8 votes

to catch up with Hung,

Of the remaining 27 votes

she needs 14 to assure a win,

hence she needs  $8+14=22$  votes.

(5pts)

**Bonus.** Explain how the ranking table of candidates A, B and C below shows that Borda's method does not satisfy the independence from irrelevant alternatives. (Recall that the property says: if A wins over B in a two-candidate race, then in a race with any additional candidates, B cannot win.)

No. of votes:	4	1	2
A	1	2	3
B	2	1	1
C	3	3	2

A Borda count with 2 candidates

has the same result as a plurality vote

so A wins over B with 4 to 3

in a race with just A, B.

Borda pts: A  $3 \cdot 4 + 2 \cdot 1 + 1 \cdot 2 = 16$

B  $3 \cdot 3 + 2 \cdot 4 = 17$

C  $2 \cdot 2 + 1 \cdot 5 = 9$