Note Title 10/6/2010

A method that does depend in part on $f(t_{i+1}, x_{i+1})$ is <u>implicit</u>.

Adams Moulton Two step Implicit Method

$x_0 = \alpha$, $x_1 = \alpha_1$

$$x_{i+1} = x_i + \frac{h}{12}\left[5f(t_{i+1}, x_{i+1}) + 8f(t_i, x_i) - f(t_{i-1}, x_{i-1})\right]$$

$i = 1, 2, \ldots, N-1$.

local truncation error

$$-\frac{1}{24} x^{(4)}(\eta_i) h^4 \text{ for some } \eta_i \in (t_{i-1}, t_{i+1})$$

Adams-Moulton Three step Implicit Method

$x_0 = \alpha_0 \quad x_1 = \alpha_1 \; , \; x_2 = \alpha_2$

$$x_{i+1} = x_i + \frac{h}{24}\Big[9f(t_{i+1}, x_{i+1}) + 19 f(t_i, x_i) - 5f(t_{i-1}, x_{i-1}) + f(t_{i-2}, x_{i-2}) \Big]$$

the local truncation error is

$$-\frac{19}{720} x^{(5)}(\eta_i) h^5 \text{ for some } \eta_i \in (t_{i-2}, t_{i+1})$$

Remarks

- the local truncation error of an $(m-1)$-step implicit method is $O(h^{m+1})$ the same as an m-step explicit method.

- Implicit multistep methods are not used alone in practice. They are used to improve approximations obtained by explicit methods.
  The combination is called a predictor - corrector method.

Stability Analysis

We examine errors that occur in the numerical solution of an initial-value problem

$$\begin{cases} x' = f(t, x) \\ x(a) = s \end{cases}$$

The exact solution is a function $x(t)$ which depends on the initial value s. Hence we write $x = x(t, s)$.

The differential equation gives rise to a family of solution curves, each corresponding to one value of the parameter s.

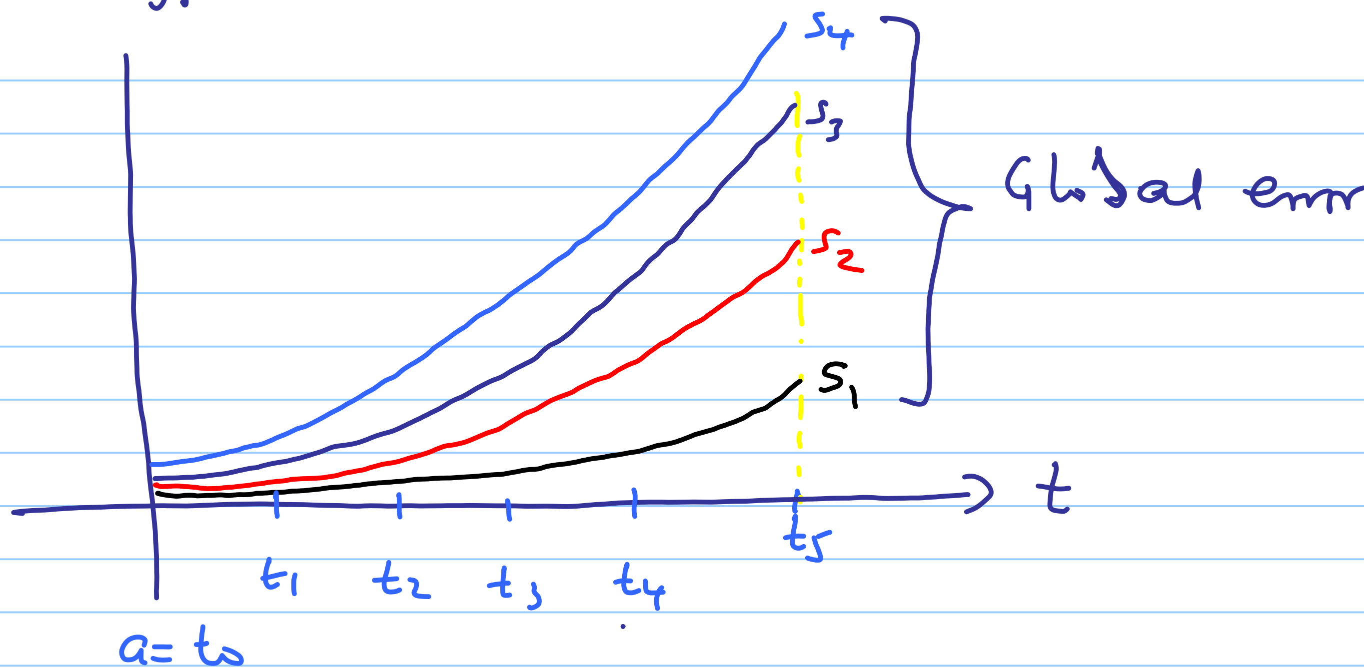
Let us consider the DE

$$\begin{cases} x' = x \\ x(a) = s \end{cases}$$

The family of solution curves is

$$x(t,s) = s e^{(t-a)}$$

that differ in their initial values $x(a) = s$.



Note that the curves diverge from one another as t increases.

An error made at the beginning has the effect of selecting the wrong curve from the family of all solution curves. The computed solution will be wrong.