October 4, 2010

Note Title 10/4/2010

#3 IVP:

$$\begin{cases} x' = x \\ x(0) = C \end{cases}$$

exact solution: $x(t) = Ce^t$

round-off errors in reading C. So we get an approximate solution

$$\tilde{x}(t) = (C+\varepsilon)e^t$$

Errors??

$$\text{error} = \tilde{x}(t) - x(t)$$

$$= (C+\varepsilon)e^t - Ce^t$$

$$\boxed{\text{error} \doteq \varepsilon e^{t}}$$

Error grows rapidly with increase in t values.

10.3 MULTISTEP METHODS

The Taylor and Runge-Kutta methods are examples of <u>one-step</u> <u>methods</u> for approximating the solution to IVP.

The methods use x_i in the approximation x_{i+1} to $x(t_i+h)$

$$\boxed{x_{i+1} \approx x(t_i+h)}$$

The prior approximations $x_0, x_1, x_2, \dots, x_{i-1}$ are not used.

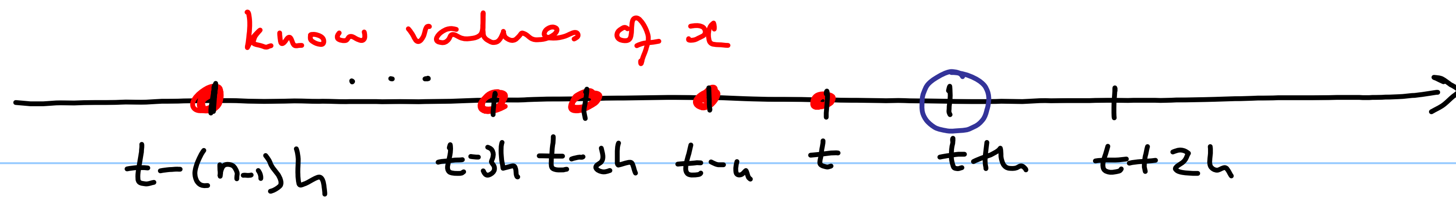
In the multistep methods some of the approximations prior to x_i are used.

Goal: to solve

$$x'(t) = f(t, x(t)), \qquad \text{(DE)}$$

Suppose that the values of the unknown function, $x(t)$ have been computed at several points to the left of t, namely

$$t, \; t-h, \; t-2h, \; t-3h, \; \dots, \; t-(n-1)h$$



we want to compute $x(t+h)$

$$\boxed{x' = f(t, x(t))}$$

Recall

$$\int_t^{t+h} x'(s)\,ds = \int_t^{t+h} f(s, x(s))\,ds$$

$$x(s)\Big|_t^{t+h} = \int_t^{t+h} f(s, x(s))\,ds$$

$$x(t+h) - x(t) = \int_t^{t+h} f(s, x(s))\,ds$$

$$x(t+h) = x(t) + \int_t^{t+h} f(s, x(s))\,ds$$

that is

$$x(t+h) = x(t) + \sum_{j=1}^{n} c_j f_j$$ For a suitable integration formula

where c_j are constants

$$f_j = f\big(t-(j-1)h,\ x(t-(j-1)h)\big)$$

$$\begin{cases} x' = f(t,x) \\ x(0) = \alpha \end{cases}$$

Adams-Bashforth Two-step Explicit Method

$x_0 = \alpha \quad , x_1 = \alpha_1$

$$x_{i+1} = x_i + \frac{h}{2}\left[3f(t_i, x_i) - f(t_{i-1}, x_{i-1})\right]$$

$i = 1, 2, \ldots, N-1$

local truncation error $= \frac{5}{12} x'''(\eta_i) h^3$ for $\eta_i \in (t_{i-1}, t_{i+1})$

$$x_2 = x_1 + \frac{h}{2}\left[3f(t_1, x_1) - f(t_0, x_0)\right]$$

$$x_3 = x_2 + \frac{h}{2}\left[3f(t_2, x_2) - f(t_1, x_1)\right]$$

$\vdots$

$x(t-h) = x(t) + -h\, x'(t)$

$= x(t) - h\, x'(\xi) \cdots$

truncation error $= x(t+h) - \left\{ x(t) + \frac{h}{2}\left[3f(t, x(t)) - f(t-h, \underline{\underline{x(t-h)}}) \right] \right.$

$$= \left(\cancel{x(t)} + hx' + \frac{h^2}{2}x'' + \frac{h^3}{3!}x''' + \cdots\cdots \right) - \cancel{x(t)}$$

$$- \frac{h}{2}\, 3f(t,x)$$

$$+ \frac{h}{2}\left(f(t,x) + \left(-h\frac{\partial}{\partial t} - hx'(\xi)\frac{\partial}{\partial x} \right) f + \left(\qquad \right)^2 f \cdots \right)$$

$$-\frac{3}{2}hf(t,x) + \frac{h}{2}f(t,x)$$

$$= -hf(t,x) = -hx'$$

Adams-Bashforth Three step Explicit method

$x_0 = \alpha, \quad x_1 = \alpha_1 \quad x_2 = \alpha_2$

$$x_{i+1} = \frac{h}{12}\left[23f(t_i, x_i) - 16f(t_{i-1}, x_{i-1}) + 5f(t_{i-2}, x_{i-2})\right]$$

where $i = 2, 3, \ldots, N-1$

local truncation error is

$$\frac{3}{8}x^{(4)}(\eta_i)h^4$$

for $\eta_i \in (t_{i-2}, t_{i+1})$.