MAT 442(01) – Spring 2013
Introduction to Numerical Analysis

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Problem Set 3
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Name : ____

1. For the given functions $f(x)$, let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct the Lagrange interpolating polynomials of degree (i) at most 1 and (ii) at most 2 to approximate $f(0.45)$, and find the actual error.
   - $f(x) = \cos x$
   - $f(x) = \sqrt{1+x}$

2. Use the Lagrange polynomial error formula to find an error bound for the approximations in the above exercise.

3. Use appropriate Lagrange interpolating polynomials to approximate $f(0.25)$ if

$$f(0.1) = 0.62049958, \quad f(0.2) = -0.28398668, \quad f(0.3) = 0.00660095, \quad f(0.4) = 0.24842440$$

4. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$, and $(2, 2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

5. The following table list s the population of the United States from 1940 to 1990. Find the Lagrange polynomial of

Year	1940	1950	1960	1970	1980	1990
Population (in thousands)	132,165	151,326	179,323	203302	226,542	249,633

degree 5 fitting this data, use this polynomial to estimate the population in the years 1930, 1965, and 2010. The population in 1930 was approximately 123,203,000. How accurate do you think your 1965 and 2010 figures are?

6. Use Newton's interpolatory divided-difference formula to construct interpolating polynomials of degrees 1, 2, and 3 for the following data. Approximate $f(0.9)$ using the polynomial.

$$f(0.6) = -0.17694460, \quad f(0.7) = 0.01375227, \quad f(0.8) = 0.22363362, \quad f(1.0) = 0.65809197$$

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Population (in thousands)	132,165	151,326	179,323	203302	226,542	249,633

7. Recall the population table Use a divided difference method to approximate each value.
   (a) The population in the year 1956.
   (b) The population in the year 1982.

8. Show that the polynomial interpolating the following data has degree 3.

x	-2	-1	0	1	2	3
f(x)	1	4	11	16	13	-4

9. Show that the Newton divided-difference polynomials
   $P(x) = 3 - 2(x+1) + 0(x+1)x + (x+1)x(x-1)$ and $Q(x) = -1 + 4(x+2) - 3(x+2)(x+1) + (x+2)(x+1)x$
   both interpolate the data

x	-2	-1	0	1	2
f(x)	-1	3	1	-1	3

   Why does the above part not violate the uniqueness property of interpolating polynomials?

10. At what step size h ought the function $\sin x$ be tabulated so that linear interpolation will produce an error of not more than $\frac{1}{2} \times 10^{-6}$?

11. Criticize the following analysis. By Taylor's formula, we have

$$\begin{aligned} f(x+h) - f(x) &= hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(\xi_1) \\ f(x-h) - f(x) &= -hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(\xi_2) \end{aligned}$$

Therefore

$$\frac{1}{h^2}[f(x+h) - 2f(x) + f(x-h)] = f''(x) + \frac{h}{6}[f'''(\xi_1) - f'''(\xi_2)]$$

The error in the approximation formula for f'' is thus $O(h)$.