Name : ______________________________

1. Verify that the polynomials

$$p(x) = 5x^3 - 27x^2 + 45x - 21, \qquad q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$$

interpolate the data below, and explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

x	1	2	3	4
y	2	1	6	47

2. Use the Lagrange interpolation process to obtain a polynomial of least degree that assumes these values:

x	1	3	-2	4	5
y	2	6	-1	-4	2

3. Consider the data

x	0	1	3	2	5
f(x)	2	1	5	6	-183

Construct the divided-difference table and using Newton's interpolation polynomial, find an approximation to $f(2.5)$.

4. It is suspected that the table below comes from a cubic polynomial. How can this be tested? Explain.

x	-2	-1	0	1	2	3
f(x)	1	4	11	16	13	-4

5. How accurately can we determine $\sin x$ by linear interpolation, given a table of $\sin x$ to ten decimal places, for x in $[0, 2]$ with $h = 0.001$?

6. Using Taylor series, establish the error term for the formula

$$f'(0) \approx \frac{1}{2h}\left[f(2h) - f(0)\right].$$

7. Criticize the following analysis. By Taylor's formula, we have

$$\begin{aligned} f(x+h) - f(x) &= hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(\xi_1) \\ f(x-h) - f(x) &= -hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(\xi_2) \end{aligned}$$

Therefore

$$\frac{1}{h^2}[f(x+h) - 2f(x) + f(x-h)] = f''(x) + \frac{h}{6}[f'''(\xi_1) - f'''(\xi_2)]$$

The error in the approximation formula for f'' is thus $O(h)$.