MAT 442(01) – SPRING 2010
Introduction to Numerical Analysis

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PROBLEM SET 1

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1. Why do the following functions not posses Taylor series expansion at $x = 0$?
   - $f(x) = \sqrt{x}$
   - $f(x) = |x|$
   - $f(x) = \arcsin(x - 1)$
   - $f(x) = \cot x$
   - $f(x) = \log x$
   - $f(x) = x^{\pi}$

2. A function f is defined by the series
$$f(x) = \sum_{k=1}^{\infty} (-1)^k \left(\frac{x^k}{k^4} \right).$$
Determine the minimum number of terms needed to compute $f(1)$ with error less than 10^{-1}.

3. Use Horner's algorithm to evaluate $p(3)$, where p is the polynomial
$$p(x) = x^4 - 4x^3 + 7x^2 - 5x - 2.$$

4. A real number x is represented approximately by 0.6032, and we are told that the relative error is at most 0.1%. What is x?

5. What is the relative error involved in rounding 4.9997 to 5.000?

6. Determine the first two nonzero terms of the series expansion about zero for the following
   - $e^{\cos x}$
   - $\sin(\cos x)$

7. Enumerate the set of numbers in the floating-point number system that have binary representation of the form
$$\pm(0.b_1b_2) \times 2^k, \text{ where } k \in \{-1, 1\}.$$

8. In the case of machine underflow, what is the relative error involved in replacing a number x by zero?

9. How can values of the function $f(x) = \sqrt{x+4} - 2$ be computed accurately when x is small?

10. What difficulty could the following assignment cause?
$$y \leftarrow 1 - \sin x.$$
Circumvent it without resorting to a Taylor series if possible.