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Name : ____

TO RECEIVE FULL CREDIT YOU MUST SHOW ALL YOUR WORK.

1. Sketch a graph of a function $f(x)$ that has the following features
   - $f(x)$ is left continuous at $x = 3$
   - $f(x)$ is not right continuous at $x = 3$
   - $f(x)$ has an infinite limit at $x = 5$
   - $\lim_{x\to 8} f(x)$ exists
   - $f(x)$ is not continuous at $x = 8$

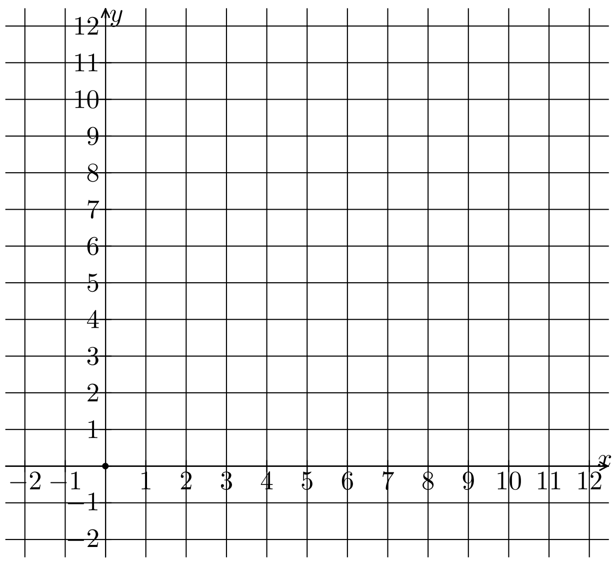

Figure 1: Graph of $f(x)$

2. Show that $\cos x = x$ has a solution in the interval $[0, 1]$.

3. Find the maximum or minimum of $y = x^2 + 6x + 2$.

4. Find the domain of the rational function $f(x) = \dfrac{x-5}{x^3 - 3x^2 + 4}$.

5. Evaluate the limit algebraically or state so if it does not exist.

- $\lim_{x\to 2} \frac{x^3 - 4x}{x - 2}$

- $\lim_{\theta\to 0} \frac{\sin(-3\theta)}{\sin(4\theta)}$

- $\lim_{\theta\to 0} \frac{1 - \cos(4\theta)}{\sin(3\theta)}$

- $\lim_{x\to 10} \frac{\sqrt{x-6} - 2}{x - 10}$

- $\lim_{t\to 0} \frac{\sin(\frac{1}{2}t)}{3t}$

6. Evaluate $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$

7. Let $f(x) = \begin{cases} x^2 + 3 & \text{for } x < 1 \\ 10 - x & \text{for } 1 \le x \le 2 \\ 6x - x^2 & \text{for } x > 2 \end{cases}$. Determine whether $f(x)$ is continuous at $x = 2$.

8. Use the limit definition to prove that $\lim_{x\to 2} x^2 = 4$