CALCULUS AND ANALYTIC GEOMETRY I - MAT 250

FALL 2008 - EXAM 4

Name :...

TO RECEIVE FULL CREDIT YOU MUST SHOW YOUR WORK. No notes or books allowed.

No. 1. (10 points) State whether each statement is **True** or **False** as stated. Provide a clear reason for your answer.

i) Assuming that $f(2) = 1$ and $f'(2) = 3$, then we can estimate $f(2.1)$.

ii) If $f(x)$ is differentiable and $f'(x) = 0$ has no solutions, then $f(x)$ has no local minima or maxima.

iii) A function that is concave down on $(-\infty, \infty)$ can have no minimum value.

iv) If $f''(c) = 0$, then f must have a point of inflection at $x = c$.

v) If f is concave up and f' changes sign at $x = c$, then f' changes sign from negative to positive at $x = c$.

No. 2. (10 points) Find the maximum and minimum values of the function (and where they occur) on the given interval.

$$y = x^3 + 3x^2 - 9x + 2, \qquad [0, 2]$$

No. 3. (10 points) Find the critical points and the intervals on which the function is increasing or decreasing, and apply the First Derivative Test to each critical point.

$$f(x) = x - \ln x, \qquad x > 0$$

No. 4. (10 points) Find the critical points of $f(x)$ and use the Second Derivative Test (if possible) to determine whether each corresponds to a local minimum or local maximum.

$$f(\theta) = \cos\theta + \sin\theta, \qquad [0, 2\pi]$$

No. 5. (15 points) Find the intervals on which f is concave up or down, the points of inflection, and the critical points, and determine whether each critical point corresponds to a local minimum or maximum (or neither).

$$f(x) = x^2(x - 4)$$

No. 6. (10 points) A box has a square base of side x and height y. Find the dimensions x, y for which the volume is 12 and the surface area is as small as possible.

No. 7. (20 points) The graph of $f'(x)$ on $[0, 6]$ is given in Figure 1.

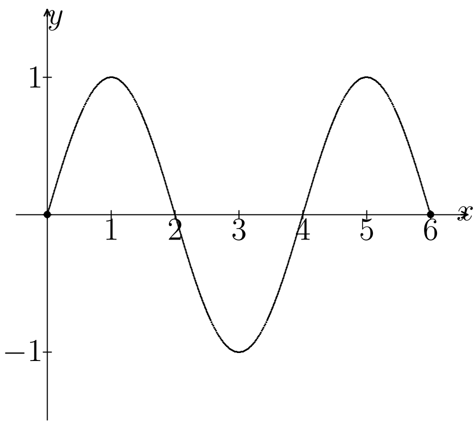

Figure 1:

i) Determine the intervals on which f is decreasing.

ii) Determine the intervals on which f is concave up.

iii) Determine the critical points, classifying each as a local maximum, minimum or neither.

iv) Determine the inflection points.

No. 8. (10 points) Evaluate the limits

a) $\displaystyle\lim_{x\to 2} \frac{2x^2 + x - 10}{2x^5 - 40x + 16}$

b) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

No. 8. (5 points) Sketch the graph of a function $f(x)$ satisfying all of the given conditions.

- $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$
- $f''(x) < 0$ for $|x| > 2$ and $f''(x) > 0$ for $|x| < 2$

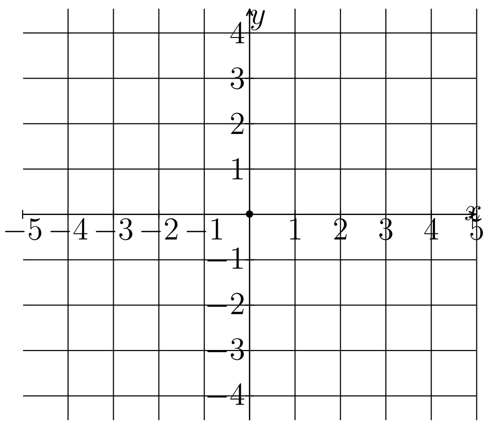

Figure 2: