

CALCULUS AND ANALYTIC GEOMETRY I - MAT 250

FALL 2008 - EXAM 3

Name :.....

TO RECEIVE FULL CREDIT YOU MUST SHOW YOUR WORK. No notes or books allowed.

No. 1. (10 points) State whether each statement is **True** or **False** as stated. Provide a clear reason for your answer.

i) Since $x^{-1} = \frac{1}{x}$ we have that $\tan^{-1} x$ and $\frac{1}{\tan x}$ are one and the same thing.

ii) $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.

iii) Since $\cosh x = \frac{e^x + e^{-x}}{2}$ we have that $\cosh 2x = \frac{e^{2x} + e^{-2x}}{4}$.

iv) $\frac{d}{dx} \tanh^{-1} x = \frac{d}{dx} \coth^{-1} x$.

v) If $f(x) = x$ then $f^{-1}(x) = x$.

No. 2. (10 points) Calculate the derivative of y with respect to x .

$$x^2y + 2xy^2 = x + y$$

No. 3. (10 points) Find $g'(-\frac{1}{2})$, where $g(x)$ is the inverse of $f(x) = \frac{x^3}{x^2 + 1}$.

No. 4. (12 points) Use logarithmic differentiation to compute the following derivatives.

i) $y = x^{\sin^{-1} x}$

ii) $y = \frac{x(x^2 + 1)}{\sqrt{x + 1}}$

No. 5. (20 points) Calculate the derivatives of the given functions:

i) $y = \arcsin(e^x)$

ii) $y = x \tan^{-1} x$

iii) $y = e^{\cos^{-1} x}$

iv) $y = \sec^{-1}(2t + 1)$

v) $f(x) = \pi^{3x+9}$

No. 6. (20 points) Find the derivative of each function

i) $y = \sinh(xe^x)$

ii) $y = \sqrt{\cosh x + 1}$

iii) $y = \sinh(\cosh x)$

iv) $y = \ln(\tanh^{-1} x)$

v) $y = \sinh^{-1}(\sqrt{x^2 + 1})$

No. 7. (18 points) A particle moves counterclockwise around the ellipse $9x^2 + 16y^2 = 25$ (Figure 1)

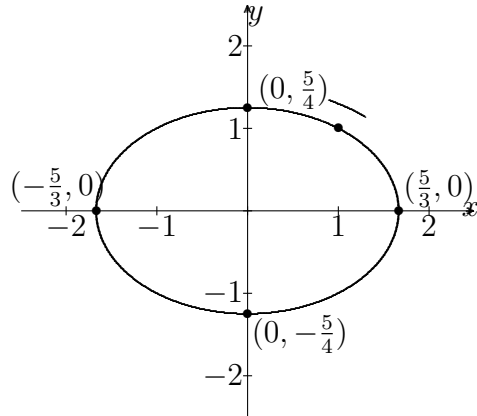


Figure 1:

- a) In which of the four quadrants is the derivative $\frac{dx}{dt}$ positive? Explain your answer.
- b) Find a relation between $\frac{dx}{dt}$ and $\frac{dy}{dt}$.
- c) At what rate is the x -coordinate changing when the particle passes the point $(1, 1)$ if its y -coordinate is increasing at a rate of 6 ft/s ?
- d) What is $\frac{dy}{dt}$ when the particle is at the top and bottom of the ellipse?