

CALCULUS AND ANALYTIC GEOMETRY I - MAT 250

FALL 2008 - EXAM 1

Name :.....

TO RECEIVE FULL CREDIT YOU MUST SHOW YOUR WORK. No notes or books allowed.

**No. 1. (20 points)** State whether each statement is **True** or **False** as stated. Provide a clear reason for your answer.

i) The lines  $y = 2x + 1$  and  $y = -2x - 4$  are perpendicular.

ii)  $|x^2 + 1|$  is not a polynomial function.

iii) Consider the function  $f$  that maps teenagers in the United States to their last names. The inverse of  $f$  does exist.

iv) Instantaneous velocity can be defined as a ratio.

v) Given any function  $f(x)$  we have that  $\lim_{x \rightarrow c} f(x) = f(c)$ .

**No. 2. (20 points)** Sketch a graph of a function  $f(x)$  that has the following features

- $f(x)$  is left continuous at  $x = 3$
- $f(x)$  is not right continuous at  $x = 3$
- $f(x)$  has an infinite limit at  $x = 5$
- $\lim_{x \rightarrow 8} f(x)$  exists
- $f(x)$  is not continuous at  $x = 8$

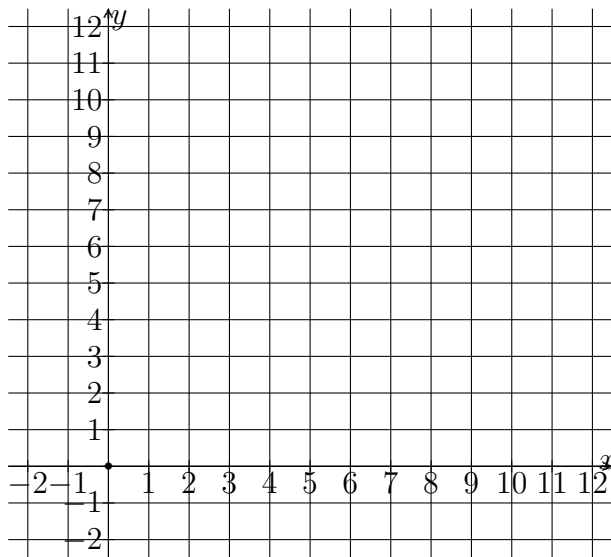


Figure 1: Graph of  $f(x)$

**No. 3. (20 points)** Show that  $\cos x = x$  has a solution in the interval  $[0, 1]$ .

**No. 4. (20 points)** Evaluate the limit algebraically or state so if it does not exist.

(i)  $\lim_{x \rightarrow 2} \frac{x^3 - 4x}{x - 2}$

(ii)  $\lim_{\theta \rightarrow 0} \frac{\sin(-3\theta)}{\sin(4\theta)}$

(iii)  $\lim_{\theta \rightarrow 0} \frac{1 - \cos(4\theta)}{\sin(3\theta)}$

(iv)  $\lim_{x \rightarrow 10} \frac{\sqrt{x - 6} - 2}{x - 10}$

**No. 5. (20 points)** Let  $f(x) = \begin{cases} x^2 + 3 & \text{for } x < 1 \\ 10 - x & \text{for } 1 \leq x \leq -2 \\ 6x - x^2 & \text{for } x > 2 \end{cases}$ . Determine whether  $f(x)$  is continuous at  $x = 2$ .

**Bonus (5 points)** Consider  $\lim_{x \rightarrow 2} \frac{1}{x}$ . Show that if  $|x - 2| < 1$  then  $|\frac{1}{x} - \frac{1}{2}| < \frac{1}{2}|x - 2|$ .