

CALCULUS AND ANALYTIC GEOMETRY I - MAT 250

FALL 2008 - Review 3

**No. I.** State whether each statement is **True** or **False** as stated. Provide a clear reason for your answer.

i)  $\tan(\arctan x) = x$ .

ii) The derivative of the inverse of a differentiable function is the inverse of the derivative of the function.

iii) If  $x$  and  $y$  are differentiable functions of  $t$  and  $y = x^2$ , then  $y'' = 2[(x')^2 + xx'']$ .

iv)  $\frac{d}{dx}b^x = (\ln b)b^x$  for  $b < 0$ .

v) Given that  $\sinh x = \frac{e^x - e^{-x}}{2}$ , we can conclude that  $\sinh x$  is never zero.

**No. II.** Calculate the derivative of  $y$  with respect to  $x$ .

- $\tan(x^2y) = x + y$

- $x^{\frac{1}{2}} + y^{-\frac{1}{2}} = 2xy$

**No. III.** Find  $g'(8)$ , where  $g(8)$  is the inverse of a differentiable function  $f(x)$  such that  $f(-1) = 8$  and  $f'(-1) = 12$ .

**No. IV.** Use logarithmic differentiation to compute the derivative of  $y = \frac{x \cos x}{(x + 1) \sin x}$

**No. V.** Find the derivative:

- $f(x) = \ln(x + e^x)$
- $G(s) = \tan^{-1}(\sqrt{s})$
- $f(x) = \ln(\csc^{-1} x)$
- $y = e^{\sec^{-1} x}$
- $g(t) = \sinh(t^2)$
- $h(y) = y \tanh(4y)$
- $g(t) = \sqrt{t^2 - 1} \sinh^{-1} t$
- $g(x) = \tanh^{-1}(e^x)$

**No. VI.**

- A bead slides down the curve  $xy = 10$ . Find the bead's horizontal velocity if its height at time  $t$  seconds is  $y = 80 - 16t^2$  cm.
- A  $6 - ft$  man walks away from a  $15 - ft$  lamppost at a speed of  $3ft/s$ . Find the rate at which his shadow is increasing in length.