CALCULUS AND ANALYTIC GEOMETRY I - MAT 250

FALL 2008 - REVIEW 1

TO RECEIVE FULL CREDIT YOU MUST SHOW YOUR WORK.

No. I. True or **False** Problems testing basic concepts. You must give clear reasons to support your choice.

No. II. For the function shown in Figure 1, find the points of discontinuity. Determine whether they are left or right continuous (or neither). Discuss any infinite discontinuities.

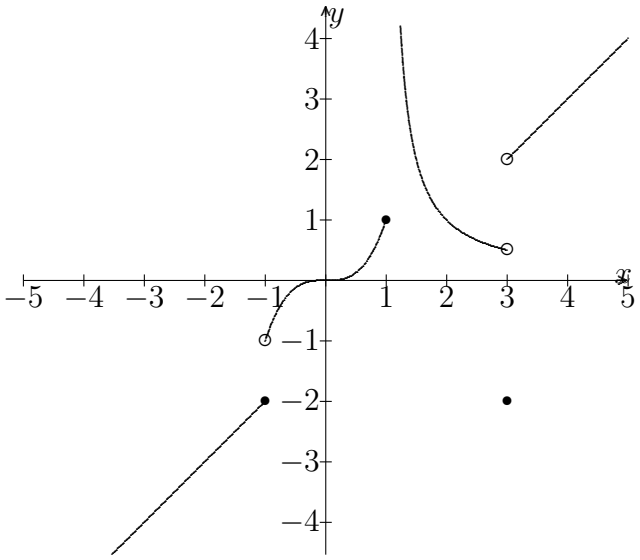

Figure 1:

III. Consider the function

$$f(x) = \begin{cases} x^3 + 1 & \text{for } -\infty < x \leq 0 \\ -x + 1 & \text{for } 0 < x \leq 2 \\ -x^2 + 10x - 15 & \text{for } x \geq 2 \end{cases} .$$

Show that $f(x)$ is continuous for $x \neq 0,\ 2$. Then compute the right and left-hand limits at $x = 1$ and 2, and determine whether $f(x)$ is left-continuous, right-continuous, or continuous at these points.

IV. Evaluate the limit or state so if it does not exist.

(i) $\lim_{x \to 8} \dfrac{x^2 - 64}{x - 8}$

(ii) $\lim_{x \to 8} \dfrac{x^2 - 64}{x - 9}$

(iii) $\lim_{x \to 1} (x - 1) \sin\left(\dfrac{\pi}{x - 1}\right)$

(iv) $\lim_{h \to 0} \dfrac{\sqrt{2 + h} - 2}{h}$

(v) $\lim_{t \to 0} \dfrac{1 - \cos t}{\sin t}$

(vi) $\lim_{t \to 0} \dfrac{\cos t - \cos^2 t}{t}$

(vii) $\lim_{x \to 4} \dfrac{\sqrt{5 - x} - 1}{2 - \sqrt{x}}$

V. Show that

- $g(t) = \dfrac{t}{t+1}$ takes on the value 0.499 for some t in $[0, 1]$.

- Show that $\tan^3\theta - 8\tan^2\theta + 17\tan\theta - 8 = 0$ has a root in $[0.5, 0.6]$.