Trigonometric Functions

6.6 Radian Measure and Applications

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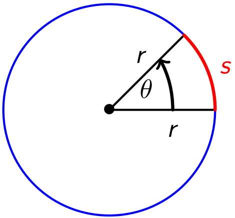
A **central angle** is an angle that has its vertex at the center of a circle.

Definition: Radian Measure

If a central angle θ in a circle wih radius r intercepts an arc on the circle of length s, then the measure of θ, in **radians** is given by

$$\theta(\text{in radian}) = \frac{s}{r}.$$

s (arc length) and r (radius) must be expressed in the same units.



Example

What is the measure (in radians) of a central angle θ that intercepts an arc of length 4 inches on a circle with radius 22 inches?

Example

What is the measure (in radians) of a central angle θ that intercepts an arc of length 20 mm on a circle with radius 100 cm?

Converting Degrees to Radians

To convert degrees to radians, multiply the degree measure by $\dfrac{\pi}{180^\circ}$.

$$\theta_r = \theta_d \frac{\pi}{180^\circ}$$

Converting Radians to Degrees

To convert radians to degrees, multiply the radian measure by $\dfrac{180^\circ}{\pi}$.

$$\theta_d = \theta_r \frac{180^\circ}{\pi}$$

Example

(a) Convert 60° to radians.

(b) Convert 340° to radians

Example

(a) Convert $\frac{11\pi}{9}$ radians to degrees

(b) Convert $\frac{4\pi}{3}$ radians to degrees

Definition: Arc Length

If a central angle θ in a circle with radius r intercepts an arc on the circle of length s, then the **arc length** s is given by

$$s = r\theta_r \qquad \theta_r \text{ is in radians}$$

$$s = r\theta_d \left(\frac{\pi}{180^\circ}\right) \qquad \theta_d \text{ is in degrees}$$

Example

In a circle with radius 6 yd, an arc in intercepted by a central angle with measure $\frac{\pi}{8}$. Find the arc length.

Definition: Area of a Circular Sector

The **area of a sector of a circle** with radius r and central angle θ is given by

$$A = \frac{1}{2} r^2 \theta_r \qquad \theta_r \text{ is in radians}$$

$$A = \frac{1}{2} r^2 \theta_d \left(\frac{\pi}{180^\circ} \right) \qquad \theta_d \text{ is in degrees}$$

Example

Find the area of the circular sector given by a radius of 3 inches and central angle $\frac{\pi}{5}$. Round your answers to three significant digits.

Definition: Linear Speed

If a point P moves along the circumference of a circle at a constant speed, then the **linear speed** v is given by

$$v = \frac{s}{t}$$

where s is the arc length and t is the time.

Example

Find the linear speed of a point that moves with constant speed in a circular motion if the point travels along the circle of arc length 12 ft in 3 min.

Example

Find the distance traveled (arc length) of a point that moves with constant speed 6.2km/hr along a circle in 4.5 hours.

Definition: Angular Speed

If a point P moves along the circumference of a circle at a constant speed, then the central angle θ that is formed with the terminal side passing through the point P also changes over some time t at a constant speed. The **angular speed** ω (omega) is given by

$$\omega = \frac{\theta}{t} \qquad \text{where } \theta \text{ is given in radian}$$

where s is the arc length and t is the time.

Example

Find the angular speed associated with rotating a central angle $\frac{3\pi}{4}$ in $\frac{1}{6}$ sec.

Relating Linear and Angular Speeds

If a point P moves at a constant speed along the circumference of a circle with radius r, then the **linear speed** v and the **angular speed** ω are related by

$$v = r\omega.$$

θ is given in radians.