

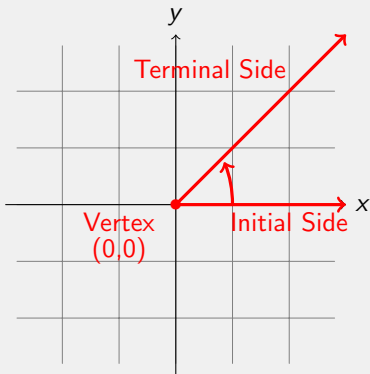
# Trigonometric Functions

## 6.4 Definition 2 of Trigonometric Functions: Cartesian Plane

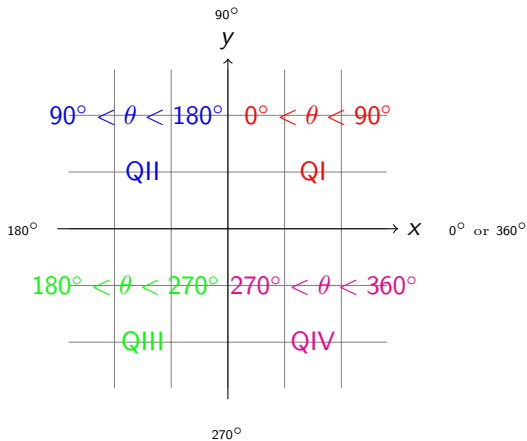
October 28, 2010

## Definition: Standard Position

An angle is said to be in **standard position** if its initial side is along the positive  $x$ -axis and its vertex is at the origin.

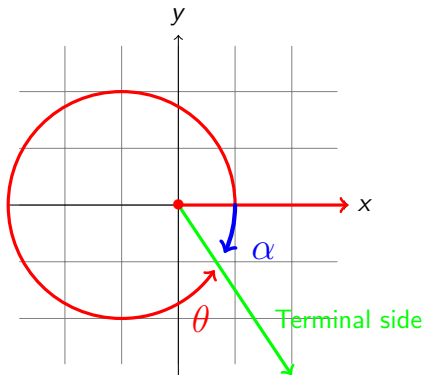


# Quadrants



## Definition: Coterminal Angles

Two angles in *standard position* with the *same terminal side* are called **coterminal angles**.



To find measures of the smallest nonnegative coterminal angles

- ▶ if the given angle is positive, subtract  $360^\circ$  repeatedly until the result is a positive angle less than or equal to  $360^\circ$ .
- ▶ If the given angle is negative, add  $360^\circ$  repeatedly until the result is a positive angle less than or equal to  $360^\circ$ .

## Example

Determine the angle of the smallest possible positive measure that is coterminal with  $945^\circ$  and  $-187^\circ$ .

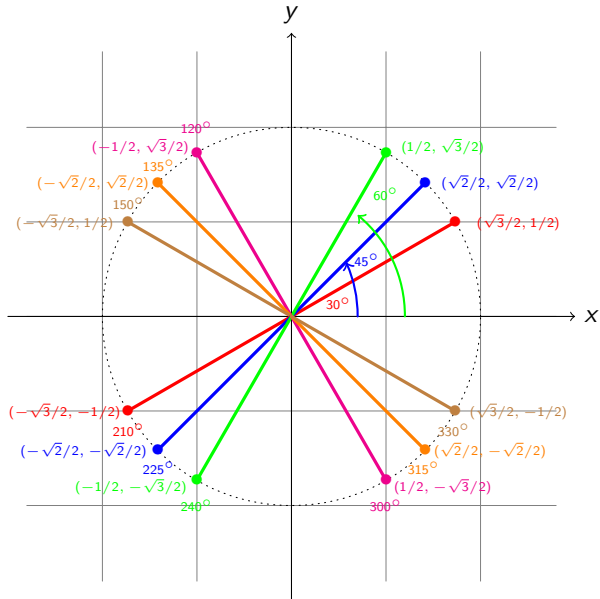
Since  $945^\circ$  is positive, subtract  $360^\circ$ .  $945^\circ - 360^\circ = 585^\circ$

Subtract  $360^\circ$  again.  $585^\circ - 360^\circ = 225^\circ$

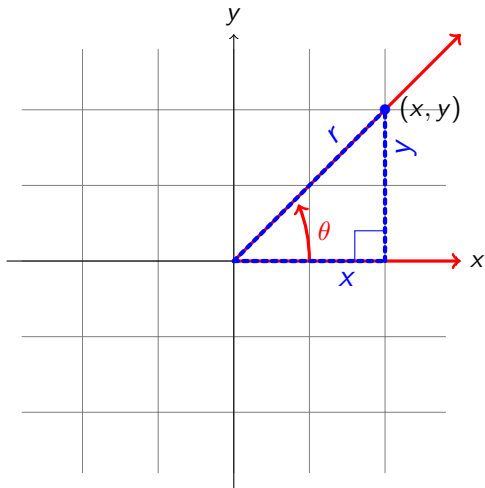
The angle with measure  $225^\circ$  is the angle with the smallest positive measure that is coterminal with the angle with measure  $945^\circ$ .

Since  $-187^\circ$  is negative, add  $360^\circ$ .  $-187^\circ + 360^\circ = 173^\circ$

# Common Angles in Standard Position



Consider an acute angle  $\theta$  in standard position and choose any point  $(x, y)$  on the terminal side of the angle as long as it is not the vertex (the origin).





## Definition 2: Trigonometric Functions

Let  $(x, y)$  be a point other than the origin on the terminal side of an angle  $\theta$  in standard position. Let  $r$  be the distance from the point  $(x, y)$  to the origin. Then the six trigonometric functions are defined as

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc \theta = \frac{r}{y} \quad (y \neq 0) \quad \sec \theta = \frac{r}{x} \quad (x \neq 0) \quad \cot \theta = \frac{x}{y} \quad (y \neq 0)$$

where  $r = \sqrt{x^2 + y^2}$ , or  $x^2 + y^2 = r^2$ . The distance  $r$  is positive:  $r > 0$ .

## Example

The terminal side of an angle  $\theta$  in standard position passes through the point  $(2, 5)$ . Calculate the values of the six trigonometric functions for angle  $\theta$ .

## Example

The terminal side of an angle  $\theta$  in standard position passes through the point  $(-4, -7)$ . Calculate the values of the six trigonometric functions for angle  $\theta$ .