# Graphs

## 2.3 Lines

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The shortest path between two points is a *straight line*.

We will discuss

- **Characteristics of lines.**
  - slope
  - intercepts
- **Types of lines:**
  - horizontal
  - vertical
  - falling
  - rising
- **Relation between lines:**
  - perpendicular
  - parallel

## Equation of a Straight line: General Form

If $A$, $B$ and $C$ are constants and $x$ and $y$ are variables, then the equation

$$Ax + By = C$$

is in **general form** and its graph is a straight line.

*Note:* $A$ or $B$ (but not both) can be zero.

## Example

Consider the equation $2x - y = -2$. It is a first degree equation, so its graph is a straight line. To graph this line

- list two solutions in a table

| x | y | (x, y) |
|---|---|---|
| -2 | -2 | (-2, -2) |
| 1 | 4 | (1,4) |

- plot those points
- use a straight edge to draw the line

The point where a line crosses, or intersects the $x$-axis is called the **x-intercept**.

The point where a line crosses, or intersects the $y$-axis is called the **y-intercept**.

## Example

The graph of the previous line has $x$-intercept (-1,0) and the $y$-intercept is (0,2).

## Example 1

Determine the $x$- and $y$-intercepts (if they exist) for the lines given.

(a) $2x + 4y = 10$

(b) $x = -2$

## Slope of a Line

A non-vertical line passing through two points $(x_1, y_1)$ and $(x_2, y_2)$ has slope, $m$, given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad \text{where } x_1 \neq x_2$$

or

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}.$$

## Example

Find the slope of the line through the points $(-2, -2)$ and $(1, 4)$.

When interpreting slope, always read the graph from *left to right*.

Lines fall into one of 4 categories:

| **Line** | **Slope** |
|---|---|
| *Rising* | *Positive* $(m > 0)$ |
| *Falling* | *Negative* $(m < 0)$ |
| *Horizontal* | *Zero* $(m = 0)$, hence $y = b$ |
| *Vertical* | *Undefined*, hence $x = a$ |

## Example 2

Sketch a line through each pair of points, classify the line as rising, falling, vertical, or horizontal, and determine its slope.

**a.** $(-1, -3)$ and $(1, 1)$

**b.** $(-3, 3)$ and $(3, 1)$

**c.** $(-1, -2)$ and $(3, -2)$

**d.** $(1, -4)$ and $(1, 3)$

## Equation of a Straight Line: Slope-Intercept Form

The **slope-intercept form** for the equation of a non-vertical line is

$$y = mx + b.$$

Its graph has slope $m$ and $y$-intercept $b$.

## Example 3

Write $2x - 3y = 15$ in slope-intercept form and graph it.

## Example 4

Find the equation of line that has slope $\frac{2}{3}$ and $y$-intercept $(0, 2)$.

## Equation of a Straight Line: Point-Slope Form

The **point-slope form** for the equation of a line is

$$y - y_1 = m(x - x_1).$$

Its graph passes through the point $(x_1, y_1)$, and its slope is $m$.

## Example 5

Find the equation of the line that has slope $-\frac{1}{2}$ and passes through the point $(-1, 2)$.

## Example 6

Find the equation of the line that passes through the points $(-2, -1)$ and $(3, 2)$.

Two distinct nonintersecting lines in a plane are *parallel*.

## Definition: Parallel Lines

Two distinct lines in a plane are **parallel** if and only if their slopes are equal.

## Example 7

Determine whether the lines $-x + 3y = -3$ and $y = \frac{1}{3}x - 6$ are parallel.

## Example 8

Find the equation of the line that passes through the point $(1, 1)$ and is parallel to the line $y = 3x + 1$.

Two *perpendicular* lines form a right angle at their point of intersection.

## Definition: Perpendicular Lines

Except for the special case of a vertical line and a horizontal line, two lines in a plane are **perpendicular** if and only if their slopes are negative reciprocals of each other.

## Example 8

Find the equation of the line that passes through the point $(3, 0)$ and is perpendicular to the line $y = 3x + 1$.

## Example 11: Application Involving Linear Equations

Suppose that your two neighbors both use the same electrician. One neighbor has a 2-hour job, which cost her $100, and another neighbor had a 3-hour job that cost him $300. Assuming that a linear equation governs the service charge of this electrician, what will your cost be for a 5-hour job?