

Equations and Inequalities

1.5 Linear Inequalities

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An example of a **linear equation** is $3x - 2 = 7$ whereas $3x - 2 \leq 7$ is an example of a **linear inequality**.

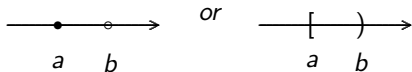
The linear equation has at most **only one solution** whereas the inequality can have a **range** or **continuum** of numbers that make the statement true.

Solutions to inequalities can be expressed in four ways:

- ▶ an inequality
- ▶ a solution set
- ▶ an interval
- ▶ a graph

Consider all real numbers **greater than or equal to** a and **less than** b . We can express them as

- ▶ **Inequality Notation:** $a \leq x < b$
- ▶ **Solution Set:** $\{x | a \leq x < b\}$
- ▶ **Interval Notation:** $[a, b)$
- ▶ **Graph/Number Line:**



Definition

Infinity (∞) is not a number. It is a symbol that means continuing indefinitely to the right of the number line. Similarly, *negative infinity* ($-\infty$) means continuing indefinitely to the left on the number line.

Example (1)

Express the following as an inequality, an interval, and a graph.

- a. x is greater than -3 .
- b. x is less than or equal to 5 .
- c. x is greater than or equal to -1 and less than 4 .
- d. x is greater than or equal to 0 and less than or equal to 4 .

In solving inequalities

- ▶ Generally we follow the same procedures used in solving linear equations.
- ▶ If you multiply or divide an inequality by a negative number, then you must change the direction of the inequality sign.

Example (3)

Solve and graph the inequality

$$5 - 3x < 23.$$

Example (4)

Solve the inequality

$$\frac{5x}{3} \leq \frac{4 + 3x}{2}.$$

Example (5)

Solve the inequality

$$-2 < 3x + 4 \leq 16.$$

Example (6)

Solve the inequality

$$1 \leq \frac{-2 - 3x}{7} < 4.$$

Express the solution set in interval notation, and graph.