

Equations and Inequalities

1.3 Quadratic Equations

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Definition

A **quadratic equation** in x is an equation that can be written in the **standard form**

$$ax^2 + bx + c = 0$$

where a , b and c are real numbers and $a \neq 0$.

In a *quadratic equation*, the variable is raised to the second power in at least one term.

Methods for solving quadratic equations:

- ▶ factoring
- ▶ the square root method
- ▶ completing the square
- ▶ the quadratic formula

The **zero product property**: If $B \cdot C = 0$, then $B = 0$ or $C = 0$ or both.

Example (1)

Solve the equation $x^2 - 6x - 16 = 0$.

Example (2)

Solve the equation $x^2 - 6x + 5 = -4$.

Example (3)

Solve the equation $2x^2 = 3x$.

Example

Solve the equation $v^2 + 7v + 6 = 0$.

Example

Solve the equation $u^2 - 2u - 24 = 0$.

Example

Solve the equation $5y^2 - 45 = 0$.

Square Root Property:

If $x^2 = P$, then $x = \pm\sqrt{P}$.

Example (4)

Solve the equation $3x^2 - 27 = 0$.

Example (5)

Solve the equation $3x^2 + 27 = 0$.

Example (6)

Solve the equation $(x - 2)^2 = 16$.

The idea behind completing the square is to transform any standard quadratic equation $ax^2 + bx + c = 0$ into the form $(x + A)^2 = B$, where A and B are constants and the left side, $(x + A)^2$, has the form of a **perfect square**. This last equation can then be solved by the square root method.

- ▶ Express the quadratic equation in the following form. $x^2 + bx = c$
- ▶ Divide b by 2 and square the result, then add the square to both sides. $x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$.
- ▶ Write the left side of the equation as a perfect square.
 $\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$
- ▶ Solve using the square root method.

Example (7)

Solve the quadratic equation $x^2 + 8x - 3 = 0$ by completing the square.

Example (8)

Solve the equation $3x^2 - 12x + 13 = 0$ by completing the square.

Quadratic Formula

If $ax^2 + bx + c = 0$, $a \neq 0$, then the solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Use the quadratic formula to solve the quadratic equation
 $x^2 - 4x - 1 = 0$.

Example

Use the quadratic formula to solve the quadratic equation $x^2 + 8 = 4x$.

Example

Use the quadratic formula to solve the quadratic equation $4x^2 - 4x + 1 = 0$.