Questions

Note Title 11/17/2010

§7.3

#5] $\tan\left(-\frac{\pi}{12}\right) = -\tan\left(\frac{\pi}{12}\right)$ $\qquad \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$$= -\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= -\left(\frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \left(\tan\frac{\pi}{3}\right)\left(\tan\frac{\pi}{4}\right)}\right)$$

$$= -\left[\frac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)}\right]$$

$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

Difference

#17) $\sin(2x)\sin(3x) + \cos(2x)\cos(3x) = \cos(2x - 3x)$

$= \cos(-x)$

$= \cos x$

#29] $\cos\alpha = -\frac{1}{3}$ $\cos\beta = -\frac{1}{4}$ α — Q III

β — Q II

$$\cos(\alpha+\beta) = \underline{\cos\alpha}\ \underline{\underline{\cos\beta}} - \sin\alpha \sin\beta$$

Now

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha + \left(-\frac{1}{3}\right)^2 = 1$$

$$\sin^2\alpha = 1 - \frac{1}{9}$$

$$\sin\alpha = \pm\sqrt{\frac{8}{9}} = \pm\frac{2\sqrt{2}}{3}$$

in Q III $\sin\alpha = \frac{-2\sqrt{2}}{3}$

$$\sin^2\beta + \cos^2\beta = 1$$

$$\sin^2\beta + \left(-\frac{1}{4}\right)^2 = 1$$

$$\sin^2\beta = 1 - \frac{1}{16}$$

$$\sin\beta = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}$$

In $\mathrm{Q}\,\overline{\underline{\text{II}}}$

$$\sin\beta = \frac{\sqrt{15}}{4}$$

$$\cos(\alpha+\beta) = \left(-\frac{1}{3}\right)\left(-\frac{1}{4}\right) - \left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{\sqrt{15}}{4}\right)$$

$$= \frac{1}{12} + \frac{\sqrt{30}}{6} \quad \text{or} \quad \frac{1+2\sqrt{30}}{12}$$

§7.4 #14

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

$$A = \frac{\pi}{8}$$

$$\frac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)} = \tan\left(2\left(\frac{\pi}{8}\right)\right)$$

$$= \tan\left(\frac{\pi}{4}\right)$$

$$= \underline{1}$$

§7.5 Half Angle Identities

Recall Double angle identities

$$\cos 2x = \cos^2 x - \sin^2 x$$

we can write as

$$\cos 2x = (1 - \sin^2 x) - \sin^2 x$$

$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

Solve for $\sin x$

$$2 \cdot \sin^2 x = 1 - \cos 2x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin x = \pm\sqrt{\frac{1 - \cos 2x}{2}}$$

Let $x = \frac{A}{2}$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos\left(2\left(\frac{A}{2}\right)\right)}{2}}$$

$$\boxed{\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{2}}}$$

Half angle identity for sine.

Similarly

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\Rightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\cos x = \pm\sqrt{\frac{\cos 2x + 1}{2}}$$

Let $x = \frac{A}{2}$

$$\boxed{\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{\cos A + 1}{2}}}$$

$$\tan^2\left(\frac{A}{2}\right) = \frac{1 - \cos A}{1 + \cos A}$$

$$= \frac{1 - \cos A}{1 + \cos A} \cdot \frac{1 + \cos A}{1 + \cos A}$$

$$= \frac{1 - \cos^2 A}{(1 + \cos A)^2}$$

$$= \frac{\sin^2 A}{(1 + \cos A)^2}$$

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A}$$

Example

$$\sin(22.5^\circ) = \sin\left(\frac{45^\circ}{2}\right)$$

$$= \pm\sqrt{\frac{1-\cos 45^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

in QI

$$= \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$

<u>Example</u>

$$\tan\left(\frac{\pi}{8}\right) = \tan\left(\frac{\pi/4}{2}\right)$$

$$= \frac{\sin(\pi/4)}{1+\cos\frac{\pi}{4}}$$ Half angle identity

$$= \frac{\sqrt{2}/2}{1+\sqrt{2}/2} = \frac{\sqrt{2}/2}{\frac{2+\sqrt{2}}{2}}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{2}{2+\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2+\sqrt{2}}$$

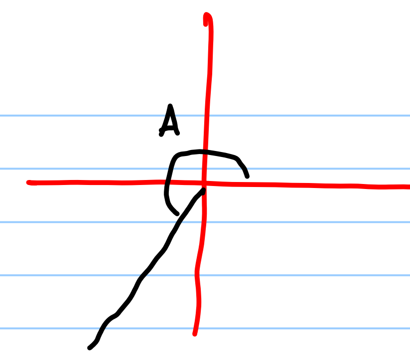
<u>Example</u>.

If $\cos x = \frac{-5}{13}$ & $\sin x < 0$, find $\cos\left(\frac{x}{2}\right)$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{\cos x + 1}{2}}$$

$$= \pm\sqrt{\frac{\frac{-5}{13} + 1}{2}}$$

$$= \pm\sqrt{\frac{\frac{8}{13}}{2}} = \pm\sqrt{\frac{4}{13}} = \pm\frac{2\sqrt{13}}{13}$$



Cos $x < 0$ & Sin $x < 0$ in QIII