§7.2 Verifying Trigonometric Identities

Note Title 11/12/2010

#14

$$\frac{\sec x}{\tan x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

#17)

$$\frac{1 - \cos^4 x}{1 + \cos^2 x} = \frac{(1 - \cos^2 x)(1 + \cos^2 x)}{1 + \cos^2 x}$$

$$= 1 - \cos^2 x \qquad \boxed{\sin^2 x + \cos^2 x = 1}$$

$$= \sin^2 x$$

Verifying Identities   LHS = RHS

Guidelines:

- Start with the more complicated side of the equation.
- Combine all sums and differences of fractions (quotients) into a single fraction (quotient)
- use the basic trigonometric identities
- use algebraic technique to manipulate

one side of the equation until the other side of the equation is achieved.

- Sometimes it is helpful to convert all trigonometric functions into sine and cosine.

<u>Example</u>

verify the identity

$$\frac{\tan x - \cot x}{\tan x + \cot x} = \sin^2 x - \cos^2 x.$$

Left hand side is more complicated

$$\frac{\tan x - \cot x}{\tan x + \cot x} = \frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

$$= \frac{\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right)}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} \quad \frac{\sin x \cos x}{\sin x \cos x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x}$$

$$= \frac{\sin^2 x - \cos^2 x}{1} \qquad \boxed{\sin^2 x + \cos^2 x = 1}$$

Done!

Example Determine whether

$$(1-\cos^2 x)(1+\cot^2 x) = 0$$

$$(1-\cos^2 x)(1+\cot^2 x) = (1-\cos^2 x)\left(1+\frac{\cos^2 x}{\sin^2 x}\right)$$

$$= \sin^2 x \cdot \left(1+\frac{\cos^2 x}{\sin^2 x}\right)$$

$$= \sin^2 x + \cos^2 x$$

$$= \underline{1} \neq 0$$

Not an identity

Verify each trigonometric identity

1. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2$$

$$= (\sin^2 x + 2\sin x\cos x + \cos^2 x) + (\sin^2 x - 2\sin x\cos x + \cos^2 x)$$

$$= 1 + 2\sin x\cdot\cos x + 1 - 2\sin x\cos x$$

$$= 2 \quad ✓$$

b) $(\csc x + 1)(\csc x - 1) = \cot^2 x$

$(\csc x + 1)(\csc x - 1) = \csc^2 x - 1$ $\boxed{1 + \cot^2 x = \csc^2 x}$

$= \cot^2 x$

c) $\tan x + \cot x = \csc x \sec x$

$\tan x + \cot x = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x}{\sin x} + \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{\cos x}$

$$= \frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x}$$

$$= \csc x \sec x \checkmark$$

d) $$\frac{2-\sin^2 x}{\cos x} = \sec x + \cos x$$

*. Suppose you took the following route

$$\frac{2-\sin^2 x}{\cos x} = \frac{2}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$= 2\sec x - \sin^2 x \sec x \;??$$

try another way

Observing RHS all depend on $\underline{\cos x}$

$$\frac{2-\sin^2 x}{\cos x} = \frac{1+1-\sin^2 x}{\cos x}$$

$$= \frac{1+\cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$= \sec x + \cos x$$

f) $$\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$$

$$\frac{\sin^2 x}{1 - \cos x} = \frac{1 - \cos^2 x}{1 - \cos x}$$

$$= \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{1 - \cos x}}$$