August 27, 2010

Note Title 8/27/2010

§1.4 #65

$$y(y-5)^3 - 14(y-5)^2 = 0$$

Common term $(y-5)^2$

$$(y-5)^2\left[y(y-5) - 14\right] = 0$$

$$(y-5)^2\left[y^2 - 5y - 14\right] = 0$$

$$(y-5)^2\left[(y-7)(y+2)\right] = 0$$

$y = 5$ or $y = 7$ or $y = -2$

$$(y-5)^2 = 0$$

$$(y-5)(y-5) = 0$$

$y=5$ $y=5$

1.6 Polynomials and Rational Inequalities

Example 1

$x^2 - x > 12$ $\qquad -\infty \longleftrightarrow +\infty$

Step 1 $\quad x^2 - x - 12 > 0$

Step 2 $\quad x^2 - x - 12 = 0$ Solve

$$(x-4)(x+3) = 0$$

Zeros: $x = 4$ or $x = -3$

Step 3:

$+$	-3	$-$	4	$+$
sign of polynomial		sign		sign

Step 4

Test value $x = -5$

$$x^2 - x - 12 = (-5)^2 - (-5) - 12 = 18 > 0$$

$$(x-4)(x+3) \qquad (-)(-)$$

$x = 0$

$$x^2 - x - 12 = 0^2 - 0 - 12 = -12 < 0$$

$$x = 5 : \; x^2 - x - 12 = 5^2 - 5 - 12 = 8 > 0$$

Step 5 & 6 : $(-\infty\ \ -3) \cup (4\ \ \infty)$

Example 2

Solve $x^2 \leq 4$

$x^2 - 4 \leq 0$

Zeros: $x^2 - 4 = 0$

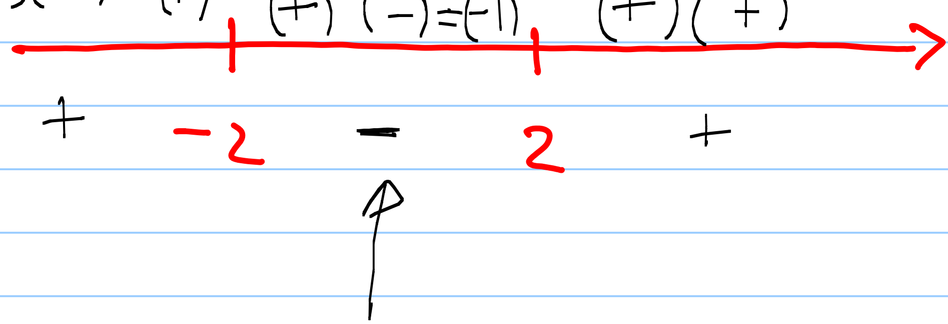
$(x+2)(x-2) = 0$

$x = -2$ or $x = 2$

$$x^2 - 4 \le 0$$

$$(x+2)(x-2) \le 0$$

$(-)(-1) = (+)$ $(+)(-) = (-1)$ $(+)(+)$



polynomial is negative on $(-2 \quad 2)$

Include endpoints because of equality

Solution $[-2 \quad 2]$.

Example 3

$$x^2 + 2x \geq -3$$

$$x^2 + 2x + 3 \geq 0$$

Zeros:

$$x^2 + 2x + 3 = 0$$

$$x : \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 - 12}}{2}$$

$$= \frac{-2 \pm \sqrt{-8}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}\,i}{2}$$

$$= -1 \pm \sqrt{2}\,i \quad \text{Not real}$$

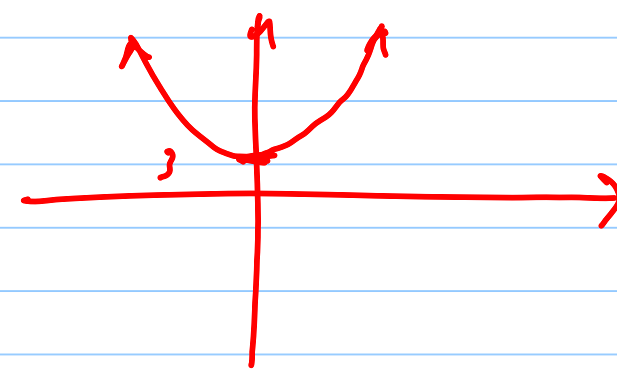
$$(-\infty, \infty)$$

The polynomial is either positive or negative on (the real line) $(-\infty \quad \infty)$

$x = 0$

$$x^2 + 2x + 3 = 0^2 + 2(0) + 3 = 3 > 0$$

Solution: $(-\infty \quad \infty)$



Example (7)

$$\frac{x-3}{x^2-4} \geq 0$$

Zeros: $x - 3 = 0$

$\boxed{x = 3}$ Numerator

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2$ or $x = 2$ Denominator

$\dfrac{(-)}{(-)(-)}$		$\dfrac{(-)}{(+)(-)}$		$\dfrac{(-)}{(+)(+)}$		$\dfrac{(+)}{(+)(+)}$
$(-)$	-2	$+$	2	$-$	3	$+$

$(-2 \;\; 2) \cup (3 \;\; \infty)$

$$\frac{x-3}{(x+2)(x-2)} \geq 0$$

the denominator cannot be equal to ± 2.

the rational expression would be undefined

when $x = 3$: $\dfrac{3-3}{(3+2)(3-2)} = 0$ $\quad \therefore$ Solution is

$(-2 \;\; 2) \cup [3 \;\; \infty)$