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1.4 Other Types of Equations

Example

$$x^{2/3} - 3x^{1/3} - 10 = 0$$

$$(a^m)^n = a^{mn}$$

$$(x^{1/3})^2 - 3(x^{1/3}) - 10 = 0 \quad = (x^{1/3})^2 - 3(x^{1/3}) - 10 = 0$$

Let $u = x^{1/3}$ (substitution)

$$u^2 - 3u - 10 = 0 \quad \text{Quadratic}$$

$$(u-5)(u+2) = 0$$

$$u = 5 \quad \text{or} \quad u = -2$$

When $u = 5$

$$x^{1/3} = 5$$

raise both sides to
power 3.

$$= 125$$

When $x = -2$

$$x^{1/3} = -2$$

The solution set is

$$(x^{1/3})^3 = (-2)^3$$

Check!!

$$x = -8$$

$$\{-8, 25\}$$

Example

$$x^{7/3} - 3x^{4/3} - 4x^{1/3} = 0$$

Recall

$$x^m \cdot x^n = x^{m+n}$$

Factor out $x^{1/3}$.

$$x^{1/3} (x^2 - 3x - 4) = 0$$

$$x^{1/3} (x-4)(x+1) = 0$$

either

$$x = 0 \quad \vee \quad x = 4 \quad \vee \quad x = -1$$

Solution set is $\{-1, 0, 4\}$.

Example

$$x^3 + 2x^2 - x - 2 = 0$$

$$x^3 + 2x^2 - x - 2 = 0$$

$$\underline{x^2(x+2)} - 1(\underline{x+2}) = 0$$

$$(x+2)(x^2 - 1) = 0$$

$$(x+2)(x+1)(x-1) = 0$$

Difference of two squares
 $(A^2 - B^2) = (A+B)(A-B)$

$$x = -2 \quad \vee \quad x = -1 \quad \vee \quad x = 1$$

1.5 Linear Inequalities

Example 1

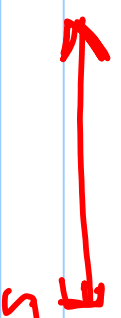
a) $x > -3$ or $-3 < x$ inequality

(-3∞) interval



b) $x \leq 5$

$(-\infty 5]$



inequality

interval

graph

$$x > -6$$

interval notation
 $(-6, \infty)$

Example

$$\frac{5x}{3} < \frac{4+3x}{2}$$

multiply both sides by LCD 6

$$6\left(\frac{5x}{3}\right) < 6\left(\frac{4+3x}{2}\right)$$

$$2(5x)$$

$$2(5x) \leq 3(4+3x)$$

$$10x \leq 12 + 9x$$

Subtract $9x$ from both sides

$$x \leq 12$$

$$(-\infty \ 12]$$

interval notation

Example 5

$$-2 < 3x + 4 \leq 16$$

Subtract 4 from each part

$$-6 < 3x \leq 12$$

divide each part by 3

$$-\frac{6}{3} < \frac{3x}{3} \leq \frac{12}{3}$$

$$\boxed{-2 < x \leq 4}$$

Example 6

$$1 \leq \frac{-2-3x}{7} < 4$$

multiply each part by 7

$$7 \leq -2-3x < 28$$

add 2 to each part

$$9 \leq -3x < 30$$

divide each by -3

$$\frac{9}{-3} \geq \frac{-3x}{-3} > \frac{30}{-3}$$

$$-3 \geq x > -10$$

Interval notation $[-10 -3]$

graph

