13.6 MEASURES OF DISPERSION

Mathematical Concepts

April 8, 2009

Measures of dispersion are used to indicate the *spread of data*. The measures of dispersion discussed are

- the range
- the standard deviation

The **range** is the difference between the highest and lowest values.

It indicates the total spread of the data.

$$\textbf{Range} = \textit{highest value} - \textit{lowest value}$$

The amount of carbohydrates, in grams, of 12 different soft drinks is given below. Determine the range of of these data.

26, 27, 31, 35, 31, 29, 24, 26, 27, 25, 30, 31

Solution:

$$Range = highest\ value - lowest\ value = 35 - 24 = 11.$$

The **standard deviation** measures how much the data *differ from the mean*.

- It is symbolized by the letter s or by the Greek letter sigma, σ.
- The s is used when the standard deviation of a *sample* is calculated.
- The σ is used when the standard deviation of the entire *population* is calculated.

What value is desirable? A very small standard deviation or a larger spread of data?

- Consider a cereal box that is to contain 8 oz of cereal. If the amount of cereal put into the box varies too much - sometimes under filling, sometimes over filling - the manufacturer would be in trouble with consumer groups and government agencies.
- Intelligence quotients (IQs) are expected to exhibit a considerable spread about the mean because everyone is different.

To Determine the Standard Deviation of a Set of Data

1. Determine the mean of the set of data.
2. Make a chart having the three columns:

$$\textit{Data} \qquad \textit{Data} - \textit{Mean} \qquad (\textit{Data} - \textit{Mean})^2$$

3. List the data vertically under the column marked *Data*.
4. Subtract the mean from each piece of data and place the difference in the $\textit{Data} - \textit{Mean}$ column.
5. Square the values in the $\textit{Data} - \textit{Mean}$ column and record these values in the $(\textit{Data} - \textit{Mean})^2$ column.
6. Determine the sum of the values in the $(\textit{Data} - \textit{Mean})^2$ column.
7. Divide the sum obtained in step 6 by $n - 1$, where n is the number of pieces of data.
8. Determine the square root of the number obtained in step 7. This number is the standard deviation, s, of the set of data.

A veterinarian in an animal hospital recorded the following life spans of selected Labrador retrievers (to the nearest year):

$$7, \quad 9, \quad 11, \quad 15, \quad 18, \quad 12$$

Determine the standard deviation of the life spans.

Solution:

First determine the mean:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{7 + 9 + 11 + 15 + 18 + 12}{6} = \frac{72}{6} = 12$$

Next construct a table with three columns.

D*ata*	**D***ata* $-$ *Mean*	$(Data - Mean)^2$
7	$7 - 12 = -5$	
9	$9 - 12 = -3$	
11	$11 - 12 = -1$	
12	$12 - 12 = 0$	
15	$15 - 12 = 3$	
18	$18 - 12 = 6$	
Sum	0	

D*ata*	**D***ata* $-$ *Mean*	$(Data - Mean)^2$
7	$7 - 12 = -5$	$(-5)^2 = (-5)(-5) = 25$
9	$9 - 12 = -3$	$(-3)^2 = (-3)(-3) = 9$
11	$11 - 12 = -1$	$(-1)^2 = (-1)(-1) = 1$
12	$12 - 12 = 0$	$(0)^2 = (0)(0) = 0$
15	$15 - 12 = 3$	$(3)^2 = (3)(3) = 9$
18	$18 - 12 = 6$	$(6)^2 = (6)(6) = 36$
Sum	0	80

Dividing the sum of $(Data - Mean)^2$ by $n - 1$

$$\frac{80}{5} = 16.$$

Finally taking the square root of this number

$$s = \sqrt{16} = 4.$$

The standard deviation is 4.

A formula for determining the standard deviation

Let x be the individual data and $\bar{x}$ the mean. Then

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$$

The following are the prices of nine stocks on the New York Stock Exchange. Determine the standard deviation of the prices.

$17, $28, $32, $36, $50, $52, $66, $74, 104

Solution:

The mean, $\bar{x}$ is

$$\begin{aligned}\bar{x} &= \frac{\Sigma x}{n} \\ &= \frac{17+28+32+36+50+52+66+74+104}{9} \\ &= \frac{459}{9} = 51\end{aligned}$$

$\mathbf{x}$	$\mathbf{x - \bar{x}}$	$\mathbf{(x - \bar{x})^2}$
17	-34	1156
28	-23	529
32	-19	361
36	-15	225
50	-1	1
52	1	1
66	15	225
74	23	529
104	53	2809
Sum	**0**	**5836**

$$
\begin{aligned}
s &= \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} \\
&= \sqrt{\frac{5836}{8}} \\
&= \sqrt{729.5} \\
&\approx 27.01
\end{aligned}
$$

The standard deviation, to the nearest tenth, is 27.01.