

## 12.1 The Nature of Probability

Mathematical Concepts

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**Definitions:**

- ▶ An **experiment** is a controlled operation that yields a set of results.
- ▶ The possible results of an experiment are called **outcomes**.
- ▶ An **event** is a sub-collection of the outcomes of an experiment.

Probability is classified as either *empirical* (experimental) or *theoretical* (mathematical).

- ▶ **empirical probability:** is the relative frequency of occurrence of an event and is determined by actual observations of an experiment.
- ▶ **theoretical probability:** is determined through a study of the possible *outcomes* that can occur for the given experiment.

The probability of an event  $E$  will be indicated by  $P(E)$ .

### **Empirical Probability (Relative Frequency)**

$$P(E) = \frac{\text{number of times event } E \text{ has occurred}}{\text{total number of times the experiment has been performed}}$$

The probability of an event is always a number between 0 and 1, inclusive.

The probability may be expressed as a decimal number or fraction.

An empirical probability of 0 indicates that the event never occurred.

An empirical probability of 1 indicates that the event has always occurred.

## Example 1

In 100 tosses of a fair coin, 44 landed heads up. Find the empirical probability of the coin landing heads up.

*Solution:* Let  $E$  be the event that the coin lands heads up.

$$\begin{aligned} \text{Then} \quad P(E) &= \frac{44}{100} \\ &= 0.44 \end{aligned}$$

## Example 2

(i) Let  $E$  be the event that weight is reduced

$$P(E) = \frac{379}{500}$$

$$= 0.758$$

A pharmaceutical company is testing a drug that is supposed to help with weight reduction. The drug is given to 500 individuals with the following outcomes.

Weight reduced	Weight unchanged	Weight increased
379	62	59

If this drug is given to an individual, find the empirical probability that the person's weight is

- i) reduced
- ii) unchanged
- iii) increased

(ii)  $E$  event that weight unchanged

$$P(E) = \frac{62}{500} = 0.124$$

$$(iii) P(\text{weight increased}) = \frac{59}{500} = 0.118$$

The **law of large numbers** states that probability statements apply in practice to a large number of trials, not to a single trial. It is the relative frequency over the long run that is accurately predictable, not individual events or precise totals.

*Example:* What does it mean to say that the probability of rolling a 2 on a die is  $\frac{1}{6}$ ?

It means that over the long run, on the average, one of every six rolls will result in a 2.