Murray State Mathematics and Statistics Colloquium
2002-2003 Academic Year

- John Schommer, University of Tennessee-Martin
  Sept. 16, 2002
  Mild and Meek Normality
  Abstract:A topological space is simply a set imposed with some additional structure, namely the designation of certain of its subsets as being "open". The real line along with its open intervals is an example of a topological space. One of the many things that topologists study is the "separation" available in an abstract topological space, i.e., the degree to which various objects can be separated by open subsets. On the real line for example, we know that any two distinct points may be separated by disjoint open intervals.

  One of the stronger separation properties that an abstract topological space can have is called normality. In a normal topological space, any two disjoint closed sets may be separated by disjoint open sets. The real line once again serves as an example. In this talk we will take a closer look at normality and two of its generalizations: mild and meek normality.

- Boris Belinskiy, University of Tennessee-Chattanooga
  Sept. 30, 2002
  Some New Developments in the Exact Control Theory
  Abstract: We study the exact controllability for a flexible elastic string fixed at the end points under an axial stretching tension. The tension is a sum of two terms, a constant tension and a slowly variable load. We say that the string is controllable if, by suitable manipulation of the input (transverse load), the output can be made to behave in some desirable way, for example, the string goes to the rest. We are looking for an exterior transverse load $g(x)f(t)$ that drives the state solution to the rest. To prove our results we apply the method of moments. This has been widely used in control theory of distributed parameter systems since the classical papers of H.O. Fattorini and D.L. Russell in the late 60s to early 70s. The problem of exact controllability is reduced to a moment problem for the control $f(t)$. The proof of controllability is based on an auxiliary basis property result.

  The results of this paper may be considered as generalization of the classical results for one-dimensional wave equation. The main difference between our problem of control and the previous problems is that the coefficient of the wave equation (tension in our model) is a function of time. As a result, the functions that substitute non-harmonic exponential functions even may not be found explicitly. This fact sufficiently complicates the analysis of controllability. To our best knowledge it is the first attempt to apply the method of moments for equations with time dependent coefficients.

- Renee Fister, Murray State University
  Oct. 7, 2002
  Optimal Control Applied to Immunotherapy
  Abstract: Optimal control techniques are applied to a cancer problem in which immunotherapy is introduced to help reduce the tumor burden. Existence and uniqueness of an optimal drug strategy is proven. This is joint work with Thalya Burden and Jon Ernstberger.

- Bruce Kessler, Western Kentucky University
  Oct. 25, 2002
  A Construction of Dilation-2 Scaling Vectors Using Fractal Interpolation Surfaces
  Abstract: Most scaling functions and scaling vectors used in wavelet applications are dilated by a factor of 2. Most people working with images use tensor products of scaling functions or vectors defined over a one-dimensional domain to do their analysis. Doug Hardin of Vanderbilt University, along with Jeff Geronimo of Georgia Tech and George Donovan currently in industry developed a dilation-3 scaling vector using fractal interpolation surfaces defined over a uniform triangulation of R^2. The speaker found the wavelets for this scaling vector, and has applied the bases to digitized images with only moderate success.

  A dilation-2 analog of the previous scaling vector has been pursued for a while, and has now been found. The speaker will present two such constructions, and show the effects of applying the bases to a digitized image. The talk will include ample background in wavelet theory and fractal surfaces and should be at a level so that the talk will be accessible to general mathematicians.

- Rick Klima, Appalachian State University (NC)
  Nov. 1, 2002
  Recent Advances in the Cryptanalysis of the Two-Message Problem
  Abstract: The Two-Message Problem involves recovering a single plaintext that has been enciphered using the Hill matrix cryptosystem with two involutory key matrices. This problem was solved for general key matrices of sizes 5x5 and smaller by Dr. Jack Levine and his students at North Carolina State University several decades ago. However, due to the extensive calculations necessary in the solution, Dr. Levine chose not to continue the cryptanalysis for larger key matrices.

  The recent development of symbolic manipulators such as Maple has allowed cryptanalysis of the Two-Message Problem to become feasible for key matrices larger than 5x5. Over the past few years, Dr. Ernest Stitzinger of North Carolina State University, Dr. Neil Sigmon of Radford University, and I have solved the problem for general key matrices of sizes 6x6 and 7x7. In this presentation, I will give a description of the Two-Message Problem, our methods and results, and how we incorporated Maple into the analysis.

- Don Hinton, University of Tennessee-Knoxville
  Nov. 16, 2002
  Sturm's 1836 Theorems on Oscillation and Comparison: Evolution of an Idea
  Abstract: We start with three of Sturm's 1836 results on oscillation and comparison for solutions of 2^{nd} order linear differential equations. We follow some threads of research stimulated by Sturm's work through not only second order equations, but also higher order differential equations and linear Hamiltonian systems. Connections with some other areas, including calculus of variations, control theory, and spectral theory of differential equations will be made.

- Ivan Ivanšić, University of Zagreb (Croatia)
  Feb. 21, 2003
  Extension dimension
  Abstract: The subject that I wish to present comes from joint work with Professor L.R. Rubin of the University of Oklahoma. It tackles a recent development in the branch of topology called Extension Theory.

  As we know, dimension theory has grown up from our intuitive notion of dimension. The first quarter of the 20th century has brought the topological theory of dimension. In fact we have several definitions of dimension. So let X be a topological space. The large inductive dimension, $IndX$, the small inductive dimension, $indX$, and the covering dimension, $dimX$, are defined purely topologically. The first two are defined inductively by means of separators, while $dimX$ is defined in terms of orders of refinements of open covers. All three agree if X is a separable metrizable space. They may differ on more general spaces. The topological definitions were soon followed by algebraic definitions of dimension, cohomological dimension, which involves algebraic topology and brings a dimension theory for each abelian group G, denoted $dim_G X$. Algebraic and topological dimensions coincide for euclidean space R^n. If $G = Z$, then for each finite dimensional compactum X, $dim_Z X$ coincides with $dimX$. In general the theories diverge significantly.

  The recent notion of *extension dimension* has its origin in theorems that characterize covering and cohomological dimensions in terms of extension of maps into certain CW complexes.Therefore for any space X and CW-complex K, one uses notation $dimX \leq K$ to mean that for each closed subset A of X and map $f : A \to K$, there exists a map $F : X \to K$ which is an extension of f. Let K and K' be CW-complexes. If it is true that for all X, $dimX \leq K$ implies that $dimX \leq K'$, then we write $K \leq K'$. This is a preorder among the CW-complexes which generates an equivalence relation $\sim$ among the CW-complexes. Then the equivalence classes $[K]$ are partially ordered and the order is induced by $\leq$. Now, for a given space X one considers:
  $$\{[L] \,|\, dimX \leq L\}$$

and asks does it have an initial element $[K]$? If it does, then $[K]$ is called the extension dimension of X.

The basic questions of the theory are the existence, representation and properties of extension dimension, and our work treats these questions.

- Kennan Shelton, Rhodes College (TN)
  March 28, 2003
  The Singled Out Game
  Abstract: The Singled Out game is based on an old MTV dating show of the same name. In our game, players attempt to guess the outcome of a coin flip by announcing their guesses so that the others can hear and in a particular sequence (not simultaneously). The first player to a given number of points wins. In the two player game, it is not hard to see that the second player has an optimal winning strategy. We use Catalan numbers to investigate the second player's winning probability as a function of the number of points required to win. We also present experimental results for the three player game.

  Comments: The Singled Out game is not mathematically challenging but presents an interesting use of Catalan numbers (related to the Ballot Problem application). I don't think that my approach to the three player game is the best possible – it and the n player game may be of interest for undergraduate research/experimentation.

- Keith Weber, Murray State University
  Apr. 11, 2003
  On Teaching, Learning, and Understanding Trigonometric Functions
  Abstract: Educational research has demonstrated that students frequently have serious difficulty in understanding trigonometric functions. In this talk, I discuss effective methods that I have employed and assessed in Math 145 to teach this topic. This presentation will have three components. I will first discuss a theory of how students come to understand mathematical concepts and will apply this theory to the specific case of trigonometric functions. I will then describe how I designed instruction to teach students about trigonometric functions based on this theory. Copies of my instructional materials will be distributed. Finally, I will report the results of a study in which I compared Math 145 students who received this novel instruction to students in a different section who received traditional instruction. The students receiving the experimental instruction demonstrated a greater understanding of trigonometric functions on paper-and-pencil tests and one-on-one interviews.